COMPUTER SYSTEMS LABORATORY

DEPARTMENTS OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
STANFORD UNIVERSITY · STANFORD, CA 94305-2192

OR DTIC

[redacted]

# PERFORMANCE OF UNIDIRECTIONAL BROADCAST LOCAL AREA NETWORKS: EXPRESS-NET AND FASNET

Michael Fine and Fouad A. Tobagi

## Technical Report No. 83-252

December 1983



This work was supported by the Defense Advanced Research Agency under Contract No. MDA 903-79-C-0201, Order No. A03717, monitored by the Office of Naval Research.

[redacted]

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | | |
|---|---|---|
| 1. REPORT NUMBER<br>Technical Report No. 83-252 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>PERFORMANCE OF UNIDIRECTIONAL BROADCAST LOCAL AREA NETWORKS: EXPRESS-NET AND FASNET | | 5. TYPE OF REPORT & PERIOD COVERED<br>Technical Report<br>6. PERFORMING ORG. REPORT NUMBER<br>83-252 |
| 7. AUTHOR(s)<br>Michael Fine and Fouad A. Tobagi | | 8. CONTRACT OR GRANT NUMBER(s)<br>MDA 903-79-C-0201<br>ARPA Order No. A03717 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Stanford Electronics Laboratories<br>Stanford University<br>Stanford, California 94305 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>Defense Advanced Research Projects Agency<br>Information Processing Techniques Office<br>1400 Wilson Ave., Arlington, VA 22209 | | 12. REPORT DATE<br>December 1983<br>13. NUMBER OF PAGES<br>83 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)<br>Resident Representative<br>Office of Naval Research<br>Durand 165<br>Stanford University | | 15. SECURITY CLASS. (of this report)<br>Unclassified<br>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Local area communication networks based on packet broadcasting techniques provide simple architectures and flexible and efficient operation. Unidirectional Broadcast Systems use a unidirectional transmission medium which, due to their physical ordering on the medium, users can access according to some efficient distributed conflict free round robin algorithm. Two systems of this type have been presented in the literature: Express-net and Fasnet. In this report we briefly describe these

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) these two. We identify three different service disciplines achievable by these systems and discuss and compare the performance of each. These systems overcome some of the performance limitations of existing random access schemes, making them particularly well suited to the high bandwidth requirements of an integrated services digital local network.

# PERFORMANCE OF UNIDIRECTIONAL BROADCAST LOCAL AREA NETWORKS: EXPRESS-NET AND FASNET.

by

Michael Fine and Fouad A. Tobagi
Computer Systems Laboratory
Department of Electrical Engineering
Stanford University, Stanford, CA 94305-2192



## Abstract

Local area communication networks based on packet broadcasting techniques provide simple architectures and flexible and efficient operation. Unidirectional Broadcast Systems use a unidirectional transmission medium which, due to their physical ordering on the medium, users can access according to some efficient distributed conflict free round robin algorithm. Two systems of this type have been presented in the literature: Express-net and Fasnet. In this report we briefly describe these two. We identify three different service disciplines achievable by these systems and discuss and compare the performance of each. These systems overcome some of the performance limitations of existing random access schemes, making them particularly well suited to the high bandwidth requirements of an integrated services digital local network.

# 1. Introduction

Local area communication networks have registered significant advances in recent years. Currently networks operating in the 1-10Mb/s range and spanning a couple of kilometers are commercially available. Although they are adequately satisfying current needs for computer communications, it appears that, in the future, there will be an increasing demand for communication resources as new system architectures (such as distributed processing) evolve and as other services such as voice, graphics and video are integrated onto the same networks.

Multiaccess broadcast bus systems have been popular since, by combining the advantages of packet switching with broadcast communication, they offer efficient solutions to the communication needs both in simplicity of topology and flexibility in satisfying growth and variability. These systems have largely used random access contention schemes such as Carrier Sense Multiple Access (CSMA). A prominent example is Ethernet [1]. Although they have proven to perform well in the environments for which they were designed, these schemes do exhibit performance limitations particularly when the channel bandwidth is high or the geographical area to be spanned is large. For example, in [2, 6] it has been shown that the maximum channel throughput for the infinite population slotted CSMA-CD scheme is given by

$$S_{\max} = \begin{cases} \dfrac{1}{1 + K\tau W/B} & (\tau W/B) \leq 0.5 \\ \dfrac{1}{(2+K)\tau W/B} & (\tau W/B) > 0.5 \end{cases} \tag{1}$$

where $K$ is a constant (in the neighborhood of 3 to 6) which depends on the particular version of the protocol, $\tau$ is the end to end propagation delay of a signal across the network in seconds, $W$ is the bandwidth of the channel in bits per second and $B$ is the number of bits in a packet.* The ratio $\tau W/B$ is referred to as the "$a$ factor". In Fig. 1, the channel

*This includes the preamble needed for synchronization.

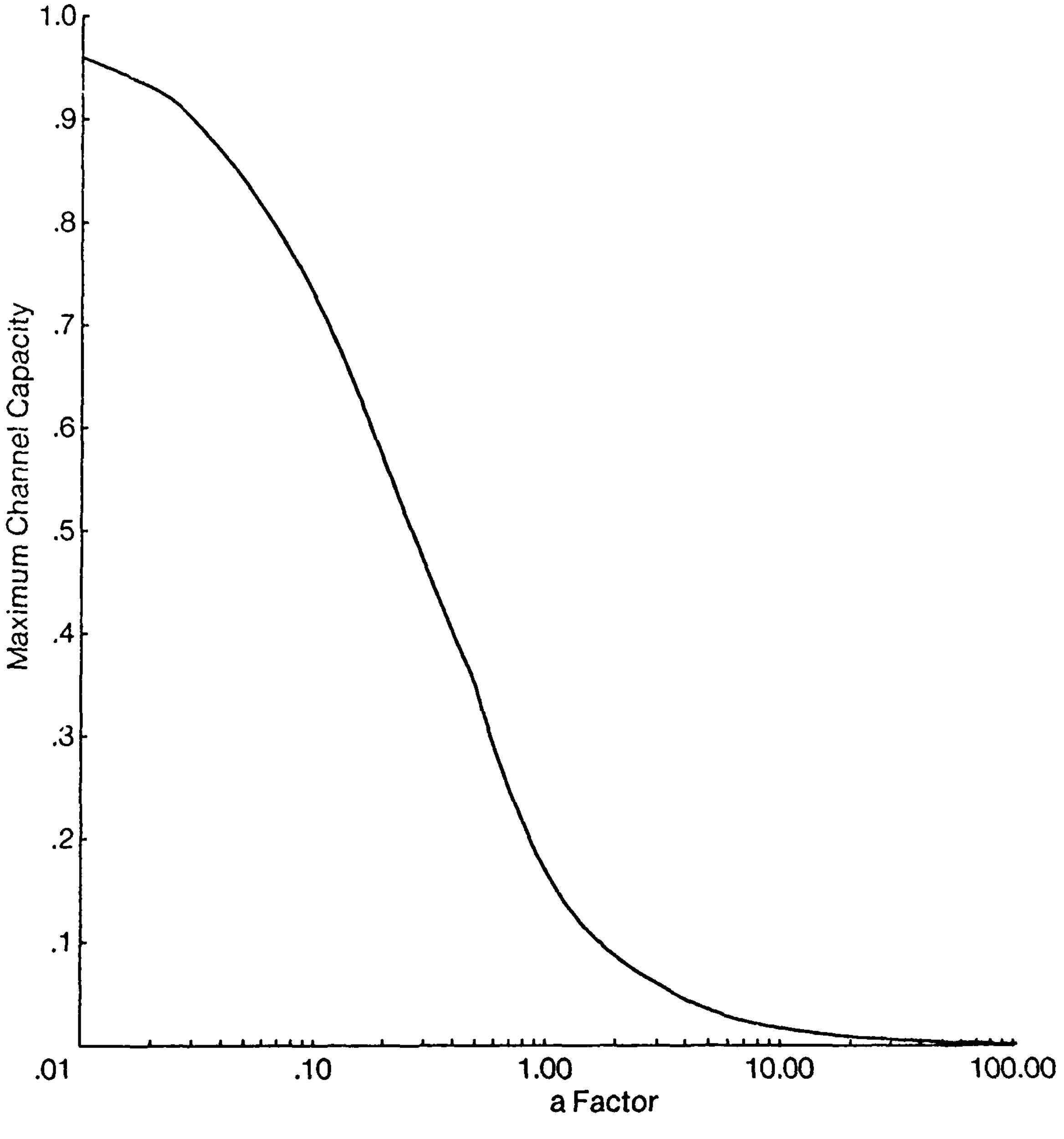


Fig. 1 Maximum channel capacity versus $a$ for CSMA-CD with infinite population.

utilization for the slotted non-persistent CSMA-CD protocol with infinite population is plotted versus $a$. We see that the channel utilization falls off rapidly for $a$ greater than about 0.1. Any effort to push the technology to higher data rates with the hope to achieve a network throughput proportional to the increase in channel bandwidth is unfortunately rewarded only by a marginal improvement.

In an attempt to overcome these limitations a new approach, also based on packet broadcasting has emerged. This type of network, which we call the Unidirectional Broadcast System (UBS) type, uses a unidirectional transmission medium on which the users contend according to some distributed conflict free round robin algorithm. We examine two recent proposals, Express-net [5, 6] and Fasnet [7, 8]. In these systems the access overhead per packet in a round is independent of both the end to end propagation delay and the number of users connected to the network. Due to this feature, these systems overcome some of the performance limitations of the random access schemes as well as earlier round robin schemes such as The Distributed Computing System [9], the UBS proposed in [10] and BRAM [12]. In this study we present quantitative results showing the performance of these networks. In section 2 we describe briefly the operation of Express-net and Fasnet with emphasis on the basic access protocol rather than on detailed functionality. As will become clear from the descriptions below, one may identify several different conflict free round robin service disciplines that can be obtained in these systems by simple modifications to the access protocols. These disciplines differ in certain aspects of the performance and it is our objective to highlight these differences. In section 3 we describe a mathematical model for the systems followed by the analysis in section 4. Finally, numerical results for the performance of these systems are discussed in section 5.

## 2. The Unidirectional Broadcast Systems Express-net and Fasnet.

The transmission medium in the unidirectional broadcast system comprises two channels which users access in order to transmit and to read the transmitted data. In Expressnet one channel, designated the *outbound* channel is used exclusively for transmitting data and the other, designated the *inbound* channel, is used exclusively for reading the transmitted data. All signals transmitted on the outbound channel are duplicated on the inbound channel thus achieving broadcast communication among the stations. In Fasnet, the transmissions on the two unidirectional channels propagate in opposite directions. Users are able to write onto and read from both. Together the two channels provide a connection between any pair of stations on the network. In both systems the asymmetry created by the unidirectional signal propagation establishes a natural ordering among the users required for the round robin access protocols described below.

## 2.1 Express-net [5, 6]

The topology of Express-net is shown in Fig. 2. In addition to writing on the outbound channel each user has the capability to sense activity on that channel due to users on the upstream side of its transmit tap. A user who has a message to transmit is said to be backlogged. Otherwise it is said to be idle. An idle user does not contend for the channel. A backlogged user operates as follows.

1. Wait for the next end of carrier on the outbound channel. (We denote this event by EOC(OUT).)

2. Immediately begin transmitting the packet and at the same time sense the outbound channel for activity from the upstream side.

3. If activity is detected from upstream then abort the transmission otherwise complete the transmission. If still backlogged go back to step 1 otherwise wait for the next packet.

Note that there is a single user which does not have to abort its transmission and hence it transmits successfully. Moreover a user who has completed the transmission of

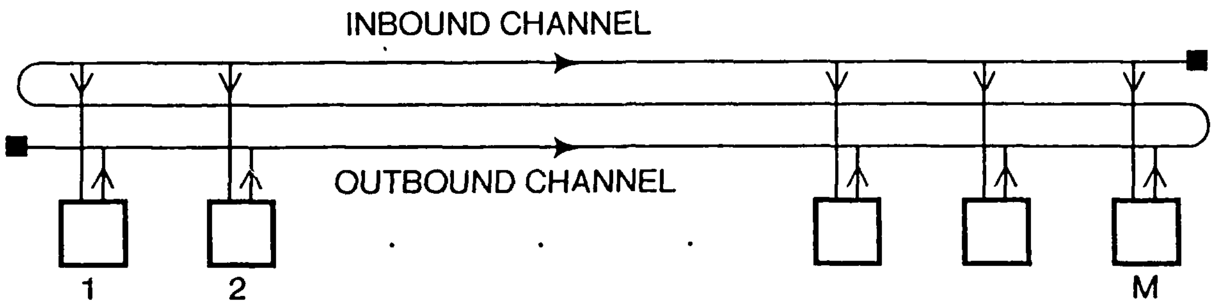


Fig. 2 The topology of Express-net.

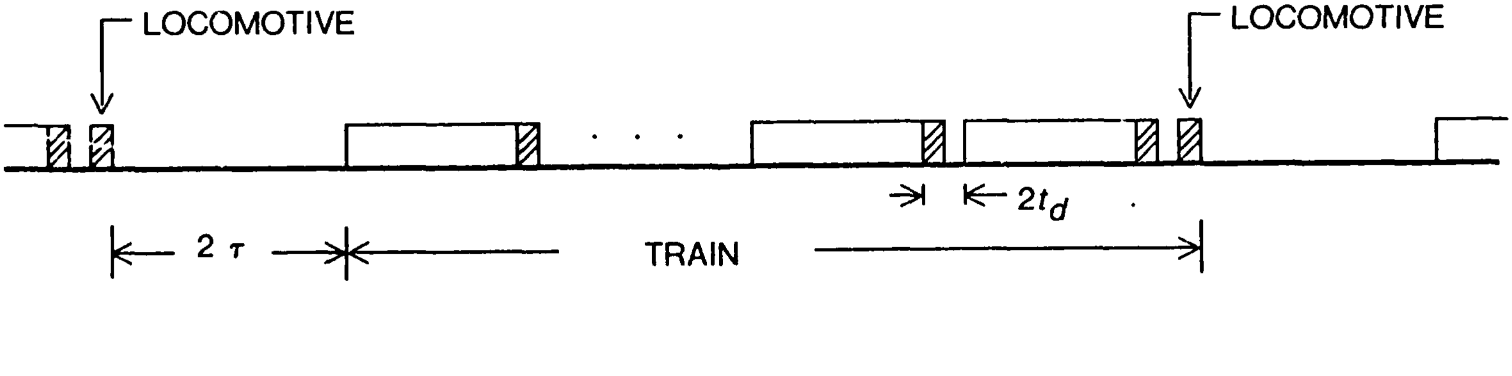


Fig. 3 Typical activity on Express-net over one cycle as seen on the inbound channel.

a packet in a given round will not encounter the event EOC(OUT) again in that round, thus guaranteeing that no user will transmit more than once in a given round. Letting $t_d$ denote the time that it takes to detect presence or absence of carrier, the gap between two consecutive packets in the same round is $t_d$ (the time required to detect EOC(OUT)), and the possible overlap at the beginning of a packet is $t_d$ (the time to detect activity due to upstream users). Thus the overhead associated with each transmission is on the order of $2t_d$ (Fig. 3).

We now describe the mechanism for initiating a new round. Define a train to be a succession of transmissions in a given round. A train is generated on the outbound channel and entirely seen on the inbound channel by all users. The end of a train on the inbound channel (EOT(IN)) is detected whenever the idle time exceeds $t_d$. Using a topology for Express-net as shown in Fig. 2, EOT(IN) will visit each user in the same order as they are permitted to transmit. Thus to start a new round, EOT(IN) is used as the synchronizing event, just as EOC(OUT) was used in the above description. Step 1 of the algorithm should be as follows.

1a. Wait for the first of the two events EOC(OUT) or EOT(IN). (Note that only one such event can occur at a given point in time.)

To avoid losing the synchronizing event EOT(IN) which happens if no packets are ready when it sweeps the inbound channel, all users (whether idle or backlogged) transmit a short burst of unmodulated carrier of duration $t_d$ whenever EOT(IN) is detected. (If the user is in the backlogged state it does so before attempting to transmit a packet.) This burst is referred to as a locomotive. If the train were to be empty, then the end of the locomotive constitutes EOT(IN). It is clear that the time separating two consecutive trains is the propagation delay between the transmit tap and the receive tap of a user, which is the same for all users. (For the topology shown in Fig. 2, this gap amounts to a round trip delay.)

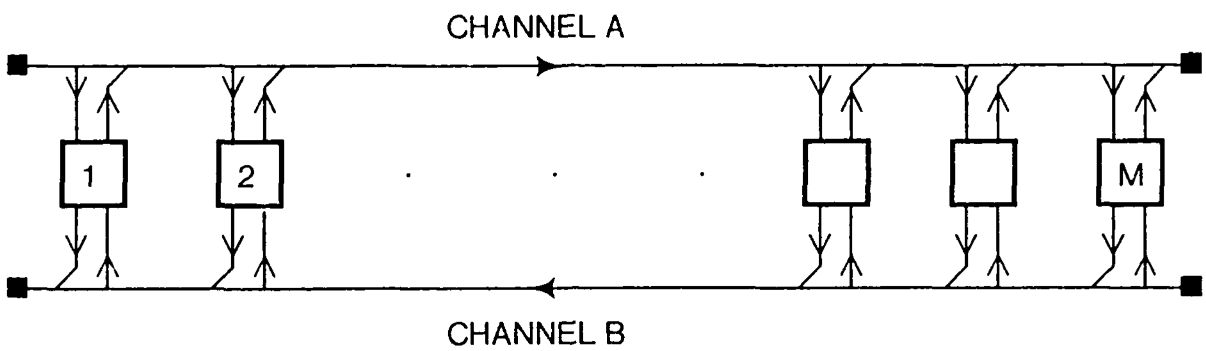


Fig. 4 The topology of Fasnet.

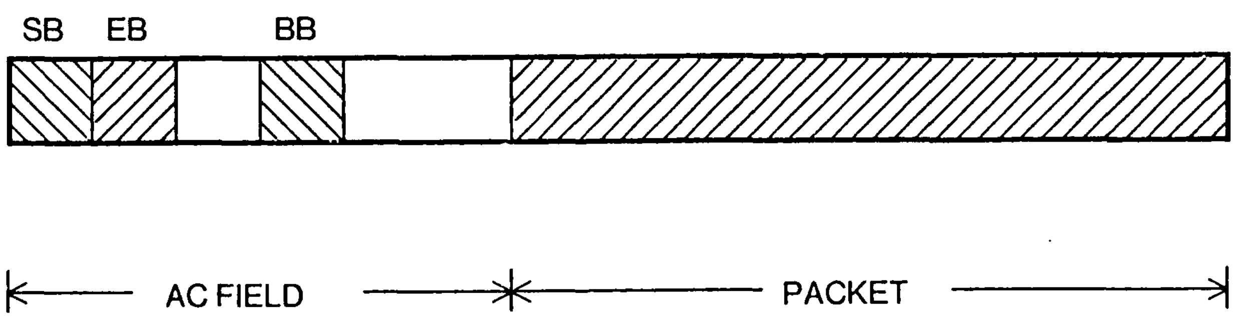


Fig. 5 Format of a slot in Fasnet.

## 2.2 Fasnet [7, 8]

The topology . Fasnet comprises two undirectional channels (A and B) where the signals propagate in opposite directions (see Fig. 4). All users can read from and write to both channels. A user wishing to send a packet will transmit on one of the channels such that the recipient is downstream from the sender. As the two channels are identical we consider events on channel A. The most upstream user (or head user) and the most downstream user (or end user) on each channel perform special functions. For channel A the head user is user 1 and the end user is user $M$. The head user transmits a clock signal which keeps the system bit synchronous.* From this clocking information users listening to the channel are able to identify fixed length slots travelling downstream. Each slot begins with an access control field (AC) which determines how and when each station may access the channel. The structure of the AC field, as shown in Fig. 5, consists of three bits. The *start* bit (SB), when set, indicates the start of a new round or cycle (SOC). The *busy* bit (BB), when set, indicates that a packet has been written into the slot. After each of these bits is a dead time which allows the user to read and process them as the slot is travelling by. The third bit, called the *end* bit (EB), is located in the dead time between the start and busy bits. This bit is used by the end user to instruct the head user *via channel B* to initiate a new cycle on channel A. We describe two different access protocols for Fasnet. The first which we call gated Fasnet is used in the most recent version of the system [7]. The second which we call non-gated Fasnet is used in an earlier version [8].

In gated Fasnet a user with no packets to transmit on channel A is said to be IDLE. Upon arrival of a packet to be transmitted on channel A (i.e, destined for a user to the right of this one) the user goes to the WAIT state. The user reads SB of every slot. When SB=1 the user goes to the DEFER state. In this state it simultaneously reads and sets

*This is to be contrasted with Express-net as described above, which is assumed to operate in asynchronous mode. In this mode, a preamble is needed for each packet for synchronization purposes at the receiver. In Fasnet a synchronization pattern is also needed but rather infrequently.

BB of each slot (setting an already set bit is assumed to have no effect). When an empty slot is detected the user writes its packet into it. It then goes to the IDLE state or WAIT state depending on whether it has more packets to transmit or not.

In the non-gated Fasnet an IDLE user is said to be either ACTIVE if it has not yet transmitted in the current round or DORMANT if it has. A DORMANT user does not attempt to access the channel. Upon arrival of a packet to an ACTIVE IDLE user, this user moves immediately to the DEFER state. It does not wait for the beginning of the next round as in the gated version. Having transmitted its packet the user becomes DORMANT and does not attempt another transmission in this round. A DORMANT user becomes ACTIVE at the beginning of the next round, i.e., when SB=1 is detected.

In both versions of Fasnet SB is set by the head user in cooperation with the end user. The end user examines all slots on channel A, decoding the status of SB and BB. Upon detecting SB=1, the end user looks for the first slot in which BB=0 (indicating that all users are IDLE or DORMANT), at which time it sets EB=1 in the next slot *on channel B*. The head user, detecting EB=1, then sets SB=1 in the next slot on channel A. Thus in the worst case the overhead in initiating a new round will be twice the end-to-end propagation delay plus twice the slot size. The additional two slots are incurred as the end user, having detected BB=0 on channel A, waits for the AC field of the next slot on channel B to set EB=1 and the head user, having detected EB=1 on channel B, waits for the next slot on channel A in order to set SB=1. It is also possible to allow the end user to set EB=1 everytime it encounters BB=0. This will result in higher throughputs since SOCs will occur at a higher frequency. However this leads to an irregular pattern of cycle lengths and unfairness among users, giving preference to upstream users. In this paper the former scheme is adopted and analyzed and it is the scheme corresponding to Fasnet.

### 2.3 The Various Service Disciplines

Clearly from the above descriptions, Express-net achieves a "conventional" round robin discipline where users are serviced in a predescribed order determined by their physical location on the network. If a user has no message when its turn comes up, it declines to transmit and then must wait for the next round before getting another turn. We refer to this type of discipline as the Non-Gated Sequential Service discipline (NGSS). The gated Fasnet system also achieves sequential service in the same physically predescribed order. In this system however, only those users who are ready at the beginning of a given round are serviced in that round. We refer to this discipline as the Gated Sequential Service discipline (GSS). In non-gated Fasnet users are also ordered according to their position on the bus; however, following a transmission, the next user to transmit is always the most upstream user who has a packet and has not yet transmitted in the current round. This discipline is referred to as the Most Upstream First Service discipline (MUFS).

Note that Express-net can be made to operate in GSS mode merely by having each user, upon generating a packet, wait for the event EOT(IN) before attempting to transmit that packet. Similarly, one could operate Fasnet in NGSS mode by allowing each user, upon generating a packet in a given round, to transmit that packet in the next empty slot as long as this user has not yet seen an empty slot go by in the current round and has not yet transmitted in the current round. Otherwise it waits for the SOC. The SOC in Fasnet and the EOT(IN) in Express-net are analogous events. In MUFS, on the other hand, the transmission of each packet is synchronized to an event which sweeps the entire population of users from the most upstream to the most downstream. Thus only Fasnet can support this discipline.

In this paper we consider only fixed length packets. In Express-net however the access protocol allows for packets of any length. In the most recent description of Fasnet [7], slots are required to be of a fixed length in order to simplify the hardware implementation. Nevertheless, in gated Fasnet, variable length packets can be accommodated simply by allowing a user to access a number of consecutive slots. This is feasible because downstream users may only transmit after the current user, and will have full knowledge of the slot usage. In non-gated Fasnet only fixed length packets equal to the size of a slot can be achieved since the order of transmissions does not correspond to the physical order; therefore the user does not know how many consecutive empty slots (if any) follow the one in which it begins to transmit.

## 3. The Model

We consider now a model which is used to analyze the performance of the three service disciplines. Consider a system of $M$ users. Each user has a single packet buffer and is either idle or backlogged. An idle user will generate a packet in a random time which is exponentially distributed with mean $1/\lambda$. A backlogged user does not generate any packets and becomes idle upon successful transmission of its buffer. This model corresponds to the case of interactive users, widely used in the past to analyze slotted ALOHA, CSMA and other access schemes. The end-to-end propagation delay of the signal travelling across the network is denoted by $\tau$. This corresponds to the propagation delay between the extreme users on one of the channels (e.g., the outbound channel on Express-net or channel A on Fasnet). The time required to transmit a packet is $T = B/W$ where $B$ is the number of bits in a packet (assumed fixed) excluding the preamble if any in Express-net and the AC field in Fasnet, and where $W$ is the bandwidth of the channel. The overhead before each transmission to determine which user gets access to the channel is denoted by $t_o$. In Fasnet $t_o$ is given by the length of the AC field. In Express-net $t_o$ is given by $2t_d$. The time required to transmit the preamble is denoted by $t_p$. In Fasnet, since the system is synchronous, $t_p = 0$. In Express-net, $t_p$ is non-zero if the system is operated asynchronously. Thus to transmit a packet of length $T$ requires a transmission period of $X = T + t_o + t_p$. The time that the channel becomes idle between rounds is called the inter-round overhead and is denoted by $Y$. In Express-net $Y = 2\tau$.* In Fasnet $Y$ must be an integral number of slots and is taken to be $Y = \lceil 2\tau/X \rceil X + X$.

In the next section we present the analysis of this model for each of the service disciplines. The performance measures derived from these analyses are the channel throughput, the expected delay incurred by a packet and the variance of this delay.

## 4. Analysis

An analysis of a loop system where users are serviced in a predescribed sequence and which fits the NGSS discipline of Express-net is given by Kaye [13] based on the work in [14]. A summary of this analysis is presented below. For the GSS discipline we present some additional definitions and both a mean value analysis and a distribution of delay analysis. Thereafter we present the analysis of the MUFS discipline which consists of a generalization of the analysis of GSS. This analysis for MUFS is exact for the case $Y \leq X$ but becomes inexact, and in fact leads to pessimistic results, for the case $Y > X$. In the discussion of numerical results in the following section, simulation is also used for MUFS when $Y > X$.

### 4.1 Analysis of the Non-Gated Sequential Service Discipline [13]

The probability that there are $n$ packet transmissions in a train, denoted by $p_n$, is given by [14]

$$p_n = p_0 \binom{M}{n} \prod_{j=0}^{n-1} \left[ e^{\lambda(jX+Y)} - 1 \right] \qquad 0 < n \leq M \tag{2}$$

*If the topology of Express-net is such that the two extreme users are collocated then the inter-round gap $Y$ is equal to $\tau$. See [6] where $p_0$ is determined by $\sum_{n=0}^{M} p_n = 1$. The probabilities $p_n$ satisfy the following recursive formula

$$\frac{p_n}{p_{n-1}} = \frac{M-n+1}{n}\left[e^{\lambda[(n-1)X+Y]} - 1\right] \tag{3}$$

Based on this distribution, Kaye derived the distribution of waiting time $\tilde{w}$, defined as the period between the moment when a packet is generated by a station and the moment when its transmission commences [13]. The expected waiting time and second moment of $\tilde{w}$ are then derived to be given by

$$E[\tilde{w}] = \frac{1}{\overline{n}} \sum_{n=1}^{M} np_n \frac{[(n-1)X+Y]e^{\lambda[(n-1)X+Y]}}{e^{\lambda[(n-1)X+Y]} - 1} - \frac{1}{\lambda} \tag{4}$$

$$E[\tilde{w}^2] = \frac{1}{\overline{n}} \sum_{n=1}^{M} np_n \frac{[(n-1)X+Y]e^{\lambda[(n-1)X+Y]}[(n-1)X+Y-2/\lambda]}{e^{\lambda[(n-1)X+Y]} - 1} + \frac{2}{\lambda^2} \tag{5}$$

where

$$\overline{n} = \sum_{n=0}^{M} np_n. \tag{6}$$

The mean and variance of packet delay are obtained by adding $X$ to $E[\tilde{w}]$ and $X^2$ to $E[\tilde{w}^2] - (E[\tilde{w}])^2$ respectively.

From the distribution $\{p_n\}$, one can also easily derive the average network throughput $S$ for a given value of $\lambda$. It is simply given by

$$S = \frac{\overline{n}T}{\overline{n}X + Y} \tag{7}$$

Note that as $\lambda \rightarrow \infty$, $\overline{n} \rightarrow M$ and the throughput reaches a maximum given by $MT/(MX + Y)$.

### 4.2 Analysis of the Gated Sequential Service Discipline

Let $n(t)$ denote the number of backlogged users at time $t$ and let $t_e^{(r)}$ denote the start of the $r^{th}$ round. Since only those users who are backlogged at $t_e^{(r)}$ can transmit in round $r$, the number of transmissions in the $r^{th}$ round is given by $n(t_e^{(r)})$. The number of backlogged users at the start of round $r+1$ depends on the length of round $r$ and the arrival of packets during this round. Hence, the number of backlogged users at $t_e^{(r+1)}$, denoted by $n(t_e^{(r+1)})$, depends only on $n(t_e^{(r)})$ and the events that occur during the $r^{th}$ round. Thus $\{n(t_e^{(r)}), r \in (-\infty, \infty)\}$ constitutes an embedded Markov process. So we can use the properties of Markov processes to derive the analytic solution for the performance of the system.

### 4.2.1 Mean Value Analysis

For the mean value analysis the state of the system at an embedded point is described sufficiently by the number of users who are backlogged at this instant. Consider two consecutive embedded points $t_e^{(r)}$ and $t_e^{(r+1)}$. The time interval $[t_e^{(r)}, t_e^{(r+1)}]$ is called a cycle. Each cycle is considered to be divided into two sub-cycles. The first is that part of the cycle where packets are being transmitted (i.e. the round). The second is that period which is the inter-round overhead. Let $\mathbf{P}$ be the transition matrix for the embedded Markov process $n(t_e^{(r)})$. The elements of $\mathbf{P}$ are denoted by $p_{ik}$ and are defined as

$$p_{ik} \triangleq \Pr\{n(t_e^{(r+1)}) = k \mid n(t_e^{(r)}) = i\} \tag{8}$$

where $n(t_e^{(r+1)})$, the state of the system at $t_e^{(r+1)}$, is merely the number of users who generate new packets during round $r$. Since those users who transmit during the round can only generate a new packet after they have transmitted the one backlogged from the previous round, the probability of generating a new packet is not the same for all users. Therefore, in computing the transition probabilities $p_{ik}$, we must account for all possible ways that $k$ out of $M$ users can become ready. To do this we use a recursive approach by considering the instants of time that correspond to the end of a transmission period.

Define the function $G(n, m, s)$ as the probability that, in a round of length $n$, $m$ users have generated new packets by the end of the $s^{th}$ transmission period of that round. We compute $G(n, m, s)$ in terms of $G(n, m', s-1)$ and this we do by computing the probability that $m - m'$ new packets are generated in the $s^{th}$ transmission period out of a possible $M-(n-s+1)-m'$. (Since $n-s+1$ users are still waiting to transmit in this round, they cannot generate new packets.) Summing over all values of $m'$ gives $G(n, m, s)$ as follows.

$$G(n,m,s) = \sum_{m'=0}^{m} \binom{M-(n-s+1)-m'}{m-m'} p(X)^{m-m'}[1-p(X)]^{M-(n-s+1)-m} \bullet G(n,m',s-1) \qquad s \neq 0 \tag{9}$$

where $p(t)$ is the probability of a single user generating a packet during an interval $t$. Since inter-arrival times are exponentially distributed with rate $\lambda$, this is given by

$$p(t) = 1 - e^{-\lambda t}. \tag{10}$$

At the beginning of the round ($s = 0$) there must be with probability 1 no new packets generated and so

$$G(n,m,0) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \tag{11}$$

Starting with these initial conditions, the recursion in (9) ends at $G(n, m, n)$, the probability that $m$ users have generated new packets by the end of the first sub-cycle.

Using (9) and considering additional arrivals during the inter-round overhead period allows us to compute the elements of the transition matrix $\mathbf{P}$.

$$p_{ik} = \sum_{j=0}^{k} G(i,j,i) \binom{M-j}{k-j} p(Y)^{k-j}[1-p(Y)]^{M-k} \tag{12}$$

Given $\mathbf{P}$ we can calculate the stationary distribution of the backlog at the embedded points and the average throughput and the average delay using results from the theory of regenerative processes. The stationary distribution is denoted by $\Pi = (\pi_0 \ldots \pi_M)$.

**Average Throughput:** Since $n(t_e^{(r)})$ is a regenerative process the channel throughput can be computed as the ratio of the expected time that the channel is busy in a cycle to the expected length of a cycle [4, 15]. Hence the expected throughput, denoted by $S$, is simply

$$S = \frac{\sum_{i=1}^{M} \pi_i T i}{\sum_{i=0}^{M} \pi_i (iX + Y)} \tag{13}$$

**Average Packet Delay:** Consider each user to be a single buffer queueing system with loss and exponential inter-arrival times. The expected delay of a packet in such a system can be computed as the difference between the expected inter-departure time and the expected inter-arrival time. Let $s_i$ denote the expected throughput of packets from user $i$. The expected inter-departure time from user $i$ is simply $1/s_i$. Hence the expected delay of a packet from user $i$ is given by

$$d_i = \frac{1}{s_i} - \frac{1}{\lambda} \tag{14}$$

Averaging over all the users gives the expected delay of a packet $D$ as

$$\begin{aligned} D &= \sum_{i=1}^{M} \frac{s_i}{S} d_i \\ &= \frac{M}{S} - \frac{1}{\lambda} \end{aligned} \tag{15}$$

where we have used the fact that $S = \sum_{i=1}^{M} s_i$.

Using Little's result, the average packet delay can also be computed as the ratio of the average backlog of packets to the average channel throughput. This approach, although significantly more involved than the one above, is nevertheless presented below. We first show how to compute the average backlog $B$.

Let $b_s(i)$ be the expected sum of the backlog over a cycle given that $n(t_e^{(r)}) = i$. This can be expressed qualitatively as

$$b_s(i) = \sum_{j=1}^{M} E[\text{time in a cycle that user } j \text{ has a packet in its buffer } \mid n(t_e^{(r)}) = i]$$

Then $B$ is computed by removing the condition on $n(t_e^{(r)})$ and dividing by the expected length of a cycle.

In order to compute the contribution to the backlog of a new arrival in an interval, say of length $t$ seconds, we need to compute the expected time of that arrival. Denote the time of this arrival by $t_a$. Then, assuming that the arrival remains in the backlog for the remainder of the interval following its arrival time, the contribution to the sum of the backlog of that arrival is $E[t - t_a]$. We define the function $u(t)$ as the value of $E[t - t_a]$ corresponding to a given generation rate $\lambda$. That is

$$u(t) = t - E[t_a \mid N(t) \neq 0] \tag{16}$$

where $N(t)$ is the number of arrivals in $[0, t]$ from a Poisson source with rate $\lambda$ and $t_a$ is the time of the first arrival. From the properties of the Poisson process it is straight forward to show that

$$E[t_a \mid N(t) \neq 0] = \frac{1}{\lambda} - \frac{te^{-\lambda t}}{1 - e^{-\lambda t}} \tag{17}$$

and so $u(t)$ becomes

$$u(t) = \frac{t}{1 - e^{-\lambda t}} - \frac{1}{\lambda} \tag{18}$$

The packets that contribute to the sum of the backlog can be separated into three groups. The first group are those that are transmitted in the current round. The packet that is transmitted in the $j^{th}$ transmission period of the round contributes an amount $jX$ to the sum of the backlog. Given that there are $i$ backlogged packets at the beginning of the cycle the total contribution to the sum of the backlog of packets of type one is simply $\sum_{j=1}^{i} jX$.

Packets of the second group are those that are generated by a user who has completed a transmission in the current round. Consider the user who has transmitted in transmission period $j$. This user will generate a new packet in the interval $[t_e^{(r)} + jX, t_e^{(r+1)}]$ with probability $p(iX + Y - jX)$. Hence the contribution to the expected sum of the backlog of this packet is $p(iX + Y - jX)u(iX + Y - jX)$.

Packets of the third group are those that arrive to a user who was idle at the beginning of the round. Such a user can generate a packet at any time in the cycle. The expected amount that such a packet will contribute to the sum of the backlog in a round with $n(t_e^{(r)}) = i$ is $p(iX + Y)u(iX + Y)$.

Summing all the contributions from these three groups gives the expected sum of the backlog over a cycle with $n(t_e^{(r)}) = i$.

$$b_s(i) = (i+1)i/2 + \left[\sum_{k=0}^{i-1} p(kX + Y)u(kX + Y)\right] + (M - i)p(iX + Y)u(iX + Y) \tag{19}$$

From (19) one can compute the expected backlog $B$. Dividing this by the expression for the expected throughput given in (13) yields the expected packet delay $D$.

$$D = \frac{\sum_{i=0}^{M} \pi_i b_s(i)}{\sum_{i=0}^{M} \pi_i iT}. \tag{20}$$

### 4.2.2 Distribution of Delay Analysis

We now derive the distribution of packet delay in order to compute the higher order moments of delay. In the distribution of delay analysis we select a single user and consider packets only from this user. We refer to this user as the tagged user. This approach will not only yield an expression for the distribution of delay but, by tagging different users on the network, will enable us to compare the performance achieved by the different users. From this we can see how a users physical location on the network can affect the quality of the service it gets from the network.

Let $N$, $1 \leq N \leq M$ denote the tagged user. As in the mean value analysis we consider the beginning of a cycle to constitute an embedded point defining an embedded Markov chain. However, in order to completely describe the state of the system at the embedded point, the state descriptor must contain information about the state of the tagged user, the number of active users upstream from the tagged user and the number of active users downstream from the tagged user. Thus we describe the state of the system at the beginning of the current round by a vector with three elements $(\delta(t_e^{(r)}), n_u(t_e^{(r)}), n_d(t_e^{(r)}))$ where $n_u(t)$ and $n_d(t)$ are the number of active users upstream and downstream from the tagged user at time $t$ respectively and $\delta(t)$ indicates the state of the tagged user at time $t$. $\delta(t)$ can take on the values 0 and 1 denoting the tagged user to be idle or busy respectively. $n_u(t)$ and $n_d(t)$ are in the range $[0, N-1]$ and $[0, M-N]$ respectively. To simplify the notation for the state descriptor we define $S(t) \triangleq (\delta(t), n_u(t), n_d(t))$.

We first compute the transition matrix $\mathbf{P}$ for the embedded Markov process $S(t_e^{(r)})$. We partition the users into three groups. The first consists of those users on the upstream side of the tagged user; the second consists of those users on the downstream side of the tagged user; the third consists of the tagged user. We compute the state transition probabilities by considering new arrivals to the system from each group separately. We now present a generalized form of the recursive function that was used in the mean value analysis. We use this generalized version to compute the state transition probabilities for the upstream and downstream groups of users.

Consider a sequence of $x$ consecutive transmissions by users from a single group. Define the function $G_Z(x, m, s \mid y)$ to be the probability that, in a transmission sequence of length $x$, $m$ users have generated new packets by the end of the $s^{th}$ transmission period in this sequence given that $y$ users had already generated new packets at the beginning of the sequence. The subscript $Z$ denotes the size of the population of users of this group. We can write this function as

$$G_Z(x, m, s \mid y) \triangleq \Pr\{n_g(t_b + sX) = x - s + m \mid n_g(t_b) = x + y\}$$

where $n_g(t)$ denotes the number of users from group $g$ who are in the backlogged state and $t_b$ is the time corresponding to the beginning of the first transmission in the sequence.

For $s > 0$ we can compute $G_Z$ recursively by considering the number of new arrivals during the $s^{th}$ transmission period.

$$G_Z(x, m, s \mid y) = \sum_{m'=y}^{m} \binom{Z - (x - s + 1) - m'}{m - m'} p(X)^{m-m'} [1 - p(X)]^{Z-(x-s+1)-m} \bullet G_Z(x, m', s - 1 \mid y) \qquad s \neq 0, s \leq x \tag{21}$$

At the beginning of the sequence there must be exactly $x + y$ users backlogged and so the boundary conditions on (21) are given by

$$G_Z(x, m, 0 \mid y) = \begin{cases} 1 & m = y \\ 0 & m \neq y \end{cases} \tag{22}$$

Since, in a given round, new arrivals to the system do not affect the order of transmissions in this round and since each user's arrival process is independent, the transition probabilities over one cycle can be computed as the product of the transition probabilities of each group of users over the cycle. Using equation (21) and conditioning on the size of the backlog of the upstream and downstream users at the beginning and end of their respective transmission sequences allows us to compute the elements of the transmission matrix $\mathbf{P}$. For $\delta(t_e^{(r)}) = 0$ we get

$p_{(0,i,j)(\beta,k,l)}$

$$
= \begin{cases} [p((i+j)X+Y)]^{\beta}[1-p((i+j)X+Y)]^{1-\beta} \\ \quad \bullet \left[\sum_{h=0}^{k} G_{N-1}(i,h,i \mid 0)\binom{N-1-h}{k-h}[p(jX+Y)]^{k-h}[1-p(jX+Y)]^{N-1-k}\right] \\ \quad \bullet \left[\sum_{h=0}^{l}\sum_{y=0}^{h}\binom{M-N-j}{y}[p(iX)]^{y}[1-p(iX)]^{M-N-j-y}G_{M-N}(j,h,j \mid y)\right. \\ \quad\quad \left.\bullet \binom{M-N-h}{l-h}[p(Y)]^{l-h}[1-p(Y)]^{M-N-l}\right] \end{cases} \tag{23}
$$

For $\delta(t_e^{(r)}) = 1$ we get

$p_{(1,i,j)(\beta,k,l)}$

$$
= \begin{cases} [p(jX+Y)]^{\beta}[1-p(jX+Y)]^{1-\beta} \\ \quad \bullet \left[\sum_{h=0}^{k} G_{N-1}(i,h,i \mid 0)\binom{N-1-h}{k-h}[p((j+1)X+Y)]^{k-h}[1-p((j+1)X+Y)]^{N-1-k}\right] \\ \quad \bullet \left[\sum_{h=0}^{l}\sum_{y=0}^{h}\binom{M-N-j}{y}[p((i+1)X)]^{y}[1-p((i+1)X)]^{M-N-j-y}G_{M-N}(j,h,j \mid y)\right. \\ \quad\quad \left.\bullet \binom{M-N-h}{l-h}[p(Y)]^{l-h}[1-p(Y)]^{M-N-l}\right] \end{cases} \tag{24}
$$

From $\mathbf{P}$ we can compute the stationary distribution at the embedded points $\Pi = (\pi_0 \ldots \pi_M)$.

Consider now an arbitrary arrival to the system in cycle $r$ from the tagged user. The delay incurred by this packet consists of two components; the delay incurred from the instant of arrival until the end of cycle $r$ and the delay incurred from the beginning of round $r+1$ until the end of the transmission of this packet. The distribution of delay is given by the convolution of these two components of delay.

Since the arrival process is memoryless we recognize that the distribution of delay of the first component of delay is given by the distribution of delay of a packet over an interval

$[0, t]$ given that the arrival occurs in this interval and that the packet remains backlogged for the remainder of the interval. Let $t_a$, $0 \leq t_a \leq t$, denote the arrival time of the packet. Then the delay incurred by the packet over the interval $[0, t]$, denoted by $D$, is $D = t - t_a$. Since inter-arrival times are exponentially distributed with mean $1/\lambda$ we can write

$$\Pr\{t_a \leq a \mid t_a \leq t\} = \frac{1}{1 - e^{-\lambda t}}[1 - e^{-\lambda a}] \qquad \text{or}$$
$$\Pr\{t_a > a \mid t_a \leq t\} = \frac{1}{1 - e^{-\lambda t}}[e^{-\lambda a} - e^{-\lambda t}]$$

The cumulative distribution function of delay is given by

$$\Pr\{D < d \mid t_a \leq t\} = \Pr\{t_a > t - d \mid t_a \leq t\} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}[e^{\lambda d} - 1]$$

Differentiating with respect to $d$ gives the probability density function

$$\mathrm{pdf}_D(d) = \lambda e^{\lambda d} \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$$

From this distribution function we can compute the Laplace transform of the distribution of delay of a packet over the interval $[0, t]$ given that the packet arrives in this interval. This distribution function denoted by $\mathcal{E}^*(t, s)$ is

$$\mathcal{E}^*(t, s) = \frac{\lambda}{\lambda - s} \frac{e^{-st} - e^{-\lambda t}}{1 - e^{-\lambda t}} \tag{25}$$

Given that this arbitrary arrival is in a round with $\mathcal{S}(t_e^{(r)}) = (\alpha, i, j)$ and $n_u(t_e^{(r+1)}) = k$ then the Laplace transform of the distribution of delay of the first component of packet delay is given by $\mathcal{E}^*(jX + Y, s)$ if $\alpha = 1$ or $\mathcal{E}^*((i + j)X + Y, s)$ if $\alpha = 0$. The second component of delay is simply $(k + 1)X$. The Laplace transform of the distribution of the total delay incurred by a packet arriving in such a round, denoted by $d^*_{(\alpha,i,j)(1,k,*)}(s)$, is given by the product of the transforms of the two distributions.

$$d^*_{(\alpha,i,j)(1,k,*)}(s) = \begin{cases} \mathcal{E}^*((i + j)X + Y, s)e^{-s(k+1)X} & \alpha = 0 \\ \mathcal{E}^*(jX + Y, s)e^{-s(k+1)X} & \alpha = 1 \end{cases} \tag{26}$$

The probability that this arbitrary packet arrives in a cycle with $S(t_e^{(r)}) = (\alpha, i, j)$ and $n_u(t_e^{(r)}) = k$ is given by

$$\varsigma_{(\alpha,i,j)(1,k,*)} \triangleq \Pr\{S(t_e^{(r)}) = (\alpha, i, j),\ n_u(t_e^{(r+1)}) = k \mid \delta(t_e^{(r+1)}) = 1\} \tag{27}$$

where by conditioning on $\delta(t_e^{(r+1)}) = 1$ we have conditioned on the event of an arrival from the tagged user in cycle $r$. Let $\Pr\{\delta(t_e^{(r+1)}) = 1\} = 1/K$ then

$$\begin{aligned} \varsigma_{(\alpha,i,j)(1,k,*)} &= \frac{\Pr\{S(t_e^{(r)}) = (\alpha, i, j),\ n_u(t_e^{(r+1)}) = k,\ \delta(t_e^{(r+1)}) = 1\}}{\Pr\{\delta(t_e^{(r+1)}) = 1\}} \\ &= K \Pr\{S(t_e^{(r)}) = (\alpha, i, j),\ n_u(t_e^{(r+1)}) = k,\ \delta(t_e^{(r+1)}) = 1\} \\ &= K \sum_{l=0}^{M-N} \Pr\{S(t_e^{(r)}) = (\alpha, i, j),\ S(t_e^{(r+1)}) = (1, k, l)\} \\ &= K \sum_{l=0}^{M-N} \pi_{(\alpha,i,j)} p_{(\alpha,i,j)(1,k,l)} \end{aligned} \tag{28}$$

and the constant $K$ can be determined from

$$\sum_{\alpha=0}^{1} \sum_{i=0}^{N-1} \sum_{j=0}^{M-N} \sum_{k=0}^{N-1} \varsigma_{(\alpha,i,j)(1,k,*)} = 1 \tag{29}$$

Using (28) to remove the conditions on $\alpha$, $i$, $j$ and $k$ in (26) we can express the Laplace transform of the distribution of delay of a packet from the tagged user as

$$D^*(s) = \sum_{\alpha=0}^{1} \sum_{i=0}^{N-1} \sum_{j=0}^{M-N} \sum_{k=0}^{N-1} \varsigma_{(\alpha,i,j)(1,k,*)} d^*_{(\alpha,i,j)(1,k,*)}(s) \tag{30}$$

By successive differentiation of (30) and letting $s = 0$ one can compute the moments of delay to any order.

## 4.3 Analysis of the Most Upstream First Service Discipline

### 4.3.1 Mean Value Analysis

The approach for this analysis is similar to that of the analysis for GSS. Again we consider two consecutive embedded points $t_e^{(r)}$ and $t_e^{(r+1)}$ and define the state of the system at the embedded points by the number of backlogged users at that instant. Let **P** be the transition matrix for the embedded Markov process $n(t_e^{(r)})$. For the MUFS discipline, the number of transmissions in cycle $r$ may be greater than $n(t_e^{(r)})$. In order to compute the elements of **P** we condition on the number of transmissions in the first sub-cycle.

$$p_{ik} = \sum_{l=i}^{M} \Pr\{n(t_e^{(r+1)}) = k \mid L = l, n(t_e^{(r)}) = i\} \Pr\{L = l \mid n(t_e^{(r)}) = i\} \tag{31}$$

where L is a random variable denoting the number of transmissions in the round. Note that by conditioning on $L$ we have removed the dependency of $n(t_e^{(r+1)})$ on $n(t_e^{(r)})$, that is

$$\Pr\{n(t_e^{(r+1)}) = k \mid L = l, n(t_e^{(r)}) = i\} = \Pr\{n(t_e^{(r+1)}) = k \mid L = l\}.$$

Let $\theta_i(l) \triangleq \Pr\{L = l \mid n(t_e^{(r)}) = i\}$ and $\phi_l(k) \triangleq \Pr\{n(t_e^{(r+1)}) = k \mid L = l\}$. Instead of enumerating all possible events over the cycle, we use recursive functions in order to compute $\theta_i(l)$ and $\phi_l(k)$. Consider a round of $L$ transmissions and the transmission period which is $s$ transmission periods from the end of the round. Define the function $F(m, s)$ as the probability of $m$ given users each generating a packet in the next $s$ transmissions. $F(m, s)$ can be computed recursively in terms of $F(m - j, s - 1)$ and the probability that there are $j$ arrivals during transmission period $s$. This gives

$$F(m, s) = \sum_{j=0}^{m} \binom{m}{j} p(X)^j [1 - p(X)]^{m-j} F(m - j, s - 1) \tag{32}$$

where $p(X)$, the probability that an idle user generates a packet within a transmission period, has been defined in equation (10). In the case $s > 0$, the number of backlogged users must be at least one to ensure that a packet transmission occurs in the next transmission period and, since there are exactly $s$ transmissions left in the sub-cycle, there can therefore be no more than $s - 1$ new packets generated in transmission period $s$. Thus $F(m, s) = 0$ for $m \geq s$, $s \neq 0$. At the end of the sub-cycle, i.e., $s = 0$, no more packets will be generated in this sub-cycle with probability one. Hence $F(0, 0) = 1$. Starting with these boundary conditions, $\theta_i(l)$ can be computed as $\binom{M-i}{l-i}F(l-i,l)$ times the probability that $M-l$ users do nothing in the sub-cycle. Note that in the case where all users are idle at the beginning of the round ($n(t_e^{(r)})=0$) then the channel remains idle until the slot immediately after some user generates a packet. Since there can be multiple arrivals during the last idle slot, the case $n(t_e^{(r)})=0$ can be computed as a convex combination of all the cases $n(t_e^{(r)})>0$. Thus $\theta_i(l)$ can be expressed as

$$\theta_i(l)=\begin{cases}[1-p(lX)]^{M-l}\binom{M-i}{l-i}F(l-i,l) & 0<i\leq M\\ \sum_{j=1}^{M}\dfrac{\binom{M}{j}p(X)^j[1-p(X)]^{M-j}}{1-[1-p(X)]^M}\theta_j(l) & i=0\end{cases} \tag{33}$$

Having conditioned on the length of the round, we can compute $\phi_l(k)$ using a similar (and simpler) recursive function to the one used in the GSS analysis. Consider the $s^{th}$ transmission period in a given round as counted from the *beginning* of the round. Define the function $G(m,s)$ as the probability that, out of the $s$ users who transmitted in the previous $s$ transmission periods, $m$ of them have generated new packets. For MUFS only those users who have transmitted in the previous $s-1$ transmission periods could have generated packets to transmit in the next round; we do not need to consider any arrivals from a user who has not yet transmitted in the current round. $G(m,s)$ can be computed recursively in terms of $G(j,s-1)$ and the probability of $m-j$ new arrivals in the $s^{th}$ transmission period. We express $G(m,s)$ as

$$G(m,s)=\sum_{j=0}^{m}\binom{s-1-j}{m-j}[p(X)]^{m-j}[1-p(X)]^{s-1-m}G(j,s-1) \tag{34}$$

where the boundary conditions are $G(m,s)=0$ for $m\geq s$ since a user can have at most one packet in its buffer waiting to be transmitted and the last user to transmit could not have generated a new one, and $G(0,1)=1$ since the number of new packets generated after the first transmission in the round is zero with probability one. $\phi_l(k)$ is given by

$$\phi_l(k) = \sum_{j=0}^{l-1} G(j,l)\binom{M-j}{k-j}[p(Y)]^{k-j}[1-p(Y)]^{M-k} \tag{35}$$

where the term $p(Y)^{k-j}[1-p(Y)]^{M-k}$ accounts for the probability that $k-j$ users generate packets during the second sub-cycle. Note that this assumes that any packets generated during the second sub-cycle remain backlogged until the next cycle. Although this assumption is exact when $Y \leq X$, it becomes inexact when $Y > X$ since, in the latter case, it may be possible for a non-dormant user to both generate and transmit a packet during the second sub-cycle. This analysis leads to pessimistic results when $Y > X$.

The elements of $\mathbf{P}$ are now computed as

$$p_{ik} = \sum_{l=\max(1,i)}^{M} \theta_i(l)\phi_l(k).$$

Given $\mathbf{P}$ we can calculate the stationary distribution of the backlog at the embedded points and the average throughput and average delay using results from the theory of regenerative processes as in the mean value analysis of the GSS discipline.

**Average Throughput:** The average channel throughput $S$ is computed as the ratio of the expected time in a cycle that the channel is carrying packets to the expected length of a cycle. The expected number of transmissions in a cycle is given by

$$\overline{L} = \sum_{i=0}^{M} \pi_i \sum_{l=\max(1,i)}^{M} \theta_i(l)l. \tag{36}$$

Hence the throughput is given by

$$S = \frac{\overline{L}T}{\pi_0 X/(1-e^{-M\lambda X}) + \overline{L}X + Y} \tag{37}$$

where the term $X/(1-e^{-M\lambda X})$ is the expected length of the idle time in a cycle where $n(t_e^{(r)}) = 0$.

**Average Packet Delay:** As in the analysis of GSS, the average delay of a packet is given by

$$D = \frac{M}{S} - \frac{1}{\lambda} \tag{38}$$

where $S$ is given by equation (37) above. Alternatively, the average delay can be computed from the average backlog and the average throughput by using Little's result.

The packets that contribute to the sum of the backlog can be separated into three groups. The first group is those packets that are transmitted during the current round. Packets of this group are those that are backlogged at $t_e^{(r)}$ and all the ones that subsequently arrive to idle users that have not yet transmitted in this round. The expected sum of the backlog due to packets in this group depends on what time during the cycle these new arrivals take place. It is extremely tedious, if not impossible to enumerate every possible combination of events. Instead we again resort to a recursive approach in order to compute the expected sum of the backlog due to these packets.

Define the function $H(m,s)$ as the expected sum of the backlog over the remaining $s$ transmission periods of the first sub-cycle given that there are $m$ non-dormant users currently backlogged. Note that $s$ represents the maximum number of remaining transmission periods. If there are no more backlogged users ($m = 0$), then the sub-cycle comes to an end. Thus we require that

$$H(0,s) = 0 \tag{39}$$

For the case $m > 0$ however, the sub-cycle does not end with the current transmission period and $H(m,s)$ is computed as the backlog over this transmission period and the remaining $s-1$ transmission periods. So

$$H(m,s) = mX + \sum_{n=m-1}^{s-1} \mu_{ms}(n)[(n-m+1)u(X) + H(n,s-1)] \tag{40}$$

where $\mu_{ms}(n)$ is the probability that $n-m+1$ new arrivals occurred out of a possible

$s - m$ and is given by

$$\mu_{ms}(n) = \binom{s-m}{n-m+1}[p(X)]^{n-m+1}[1-p(X)]^{s-n-1} \tag{41}$$

The expected sum of the backlog, conditioned on $n(t_e^{(r)}) = i$, due to packets of the first group is simply given by $H(i, M)$.

Packets from the second group are those that are generated by a dormant user and hence only transmitted in the round following the current one. The contribution to $b_s(i)$ of any dormant user is the expected length of time from the instant that the new packet is generated to the end of the cycle multiplied by the probability that a packet is generated by this user. Consider the $j^{th}$ user to transmit in a round of length $L$. This user will generate a new packet in the interval $[t_e^{(r)} + jX, t_e^{(r+1)}]$ with probability $p((L-j)X+Y)$. Hence the contribution to the expected sum of the backlog of this packet, conditioned on the length of a round, is $p((L-j)X+Y)u[(L-j)X+Y]$. Summing contributions from all such packets and removing the condition on the length of the round gives the expected sum of the backlog due to packets from the second group.

Packets of the third group are those that arrive to an active user and are transmitted in round following the current one. This group includes only packets generated in the inter-round overhead period. Given that the round is of length $L$, there are $M - L$ users who can contribute a packet to this group. The contribution of each of these to the expected sum of the backlog is $p(Y)u(Y)$. Summing these contributions and removing the condition on the length of the round gives the expected sum of the backlog due to packets from the third group.

For a cycle with $n(t_e^{(r)}) = 0$, the round starts at the beginning of the first slot after the first arrival. In this case $b_s(0)$ can be expressed in terms of $b_s(i)$, $i > 0$, by adding the additional contribution to the backlog of those arrivals in the last idle slot. Hence $b_s(i)$ is given by

$$b_s(i) = \begin{cases} H(i,M) + \sum_{l=i}^{M} \theta_i(l) \sum_{j=0}^{l-1} p(jX+Y)u(jX+Y) + \sum_{l=1}^{M} \theta_i(l)(M-l)p(Y)u(Y) \\ \hfill 1 \leq i \leq M \\ \sum_{j=1}^{M} \frac{\binom{M}{j} p(X)^j [1-p(X)]^{M-j}}{1-[1-p(X)]^M} [ju(X) + b_s(j)] \hfill i = 0 \end{cases} \tag{42}$$

From equation (42) we can calculate the expected backlog as

$$B = \frac{\sum_{i=0}^{M} \pi_i b_s(i)}{\pi_0 X/(1-e^{-M\lambda X}) + \bar{L}X + Y} \tag{43}$$

From $B$ and the expression for the throughput given in (37), the average packet delay $D$ is expressed as

$$D = \frac{\sum_{i=0}^{M} \pi_i b_s(i)}{\bar{L}T}. \tag{44}$$

### 4.3.2 Distribution of Delay Analysis

In the GSS analysis we considered a single user to be tagged and then considered packets from this user. To compute the transition probabilities of the embedded Markov process, we divided the users into three groups and calculated the transition probabilities of each group independently. This was possible since the transmissions of the users in each group are contiguous due to the predetermined order in which the users are allowed to transmit in GSS. For MUFS, the order of transmissions varies according to the arrival patern of packets to the users. Therefore we are unable to use this approach to compute the distribution of delay in the general case without resorting to an extremely cumbersome state descriptor. However, if we consider that the tagged user is either the most upstream user (user 1) or the most downstream user (user $M$), the size of one of the three groups is always *zero* and the analysis becomes more easily manageable. In this section we present the analysis for the case where one of the end users is the tagged one.

The Markov process that we are now required to analysis is sufficiently described by the number of users in each group at the embedded points. We define the state descriptor as $S(t_e^{(r)}) \triangleq (\delta(t_e^{(r)}), n(t_e^{(r)}))$ where $\delta(t_e^{(r)})$ denotes the state of the tagged user and $n(t_e^{(r)})$ is the number of backlogged users excluding the tagged one. In order to compute the transition matrix $\mathbf{P}$, we condition on both the number of transmissions in a round and the transmission period in which the tagged user transmits. The elements of $\mathbf{P}$ are given by

$$p_{(\alpha,i)(\gamma,k)} = \sum_{l=\alpha+i}^{M} \sum_{y=0}^{l} \Pr\{S(t_e^{(r+1)}) = (\gamma, k) \mid L = l,\ TP_t = y,\ S(t_e^{(r)}) = (\alpha, i)\} \bullet \Pr\{L = l,\ TP_t = y \mid S(t_e^{(r)}) = (\alpha, i)\} \tag{45}$$

where $L$ is a random variable denoting the number of transmissions in the current round and $TP_t$ is a random variable denoting the transmission period, counted from the beginning of the round, in which the tagged user transmits a packet. We adopt the convention $TP_t = 0$ to indicate that the tagged user does not transmit in the current round. Note that conditioning on $L$ and $TP_t$ removes the dependency of $S(t_e^{(r+1)})$ on $S(t_e^{(r)})$. Thus

$$\Pr\{S(t_e^{(r+1)}) = (\gamma, k) \mid L = l,\ TP_t = y,\ S(t_e^{(r)}) = (\alpha, i)\} = \Pr\{S(t_e^{(r+1)}) = (\gamma, k) \mid L = l,\ TP_t = y\} \tag{46}$$

Let $\theta_{(\alpha,i)}(l, y) \triangleq \Pr\{L = l, TP_t = y \mid S(t_e^{(r)}) = (\alpha, i)\}$ and $\phi_{(l,y)}(\gamma, k) \triangleq \Pr\{S(t_e^{(r+1)}) = (\gamma, k) \mid L = l,\ TP_t = y\}$. $\theta$ and $\phi$ are now computed using recursive functions in a similar way to that used in the mean value analysis of MUFS. In order to compute $\theta$ we must distinguish which user is tagged.

**Most upstream user tagged:** Consider a round of length $L$. For the transmission period that is $s$ transmission periods away from the end of the round where $0 \leq s \leq L$, define the function $f(x, m, s)$ as the joint probability of the tagged user transmiting in the $x^{th}$ transmission period *from the end of the round* (i.e. arriving in transmission period $x+1$) and $m$ given users *excluding* the tagged user each generating a packet in the next $s$ transmissions. Since $x$ is counted from the end of the round and $TP_t$ is counted from the beginning of the round we have the relationship

$$TP_t = L - x + 1. \tag{47}$$

For the case $s = 0$ we must have

$$\begin{array}{ll} f(x,0,0) = 1 & \text{and} \\ f(x,m,0) = 0 & \text{for} \quad m > 0 \end{array} \tag{48}$$

since, when the sub-cycle has ended, there must be with probability one no backlogged users. For the case $s \neq 0$ we already have at least one backlogged user (otherwise the sub-cycle would end which implies that $s = 0$) and thus cannot allow more than $s-1$ of the remaining users to generate new packets. This requires that

$$f(x,m,s) = 0 \qquad \begin{array}{l} \text{for } m \geq s,\ s \leq x,\ s \neq 0 \\ \text{or } m \geq s-1,\ s > x \end{array} \tag{49}$$

The distinction between the case $s \leq x$ and $s > x$ is made because the number of given users $m$ does not include the tagged one. For the case where $m < s$, $s \leq x$, $s \neq 0$ or $m < s-1$, $s > x$, $f(x, m, s)$ can be computed recursively in terms of $f(x, m', s-1)$ by considering the number of new packets that are generated in the next transmission period. Ignoring the tagged user we can have up to $m$ new arrivals.

$$f(x,m,s) = \begin{cases} \displaystyle\sum_{j=0}^{m} \binom{m}{j} [p(X)]^j [1-p(X)]^{m-j} f(x, m-j, s-1) & \begin{array}{l} m < s,\ s \leq x,\ s \neq 0 \\ \text{or } m < s-1,\ s > x \end{array} \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

$\theta_{(\alpha,i)}(l,y)$ is computed as the product of $f(l-y+1, l-i-\alpha, l)$, the number of ways of choosing $l-i-1$ users out of a possible $M-i-1$ (since $i$ users and the tagged user are already given) and the probability that the remaining $M-l$ users do not generate any packets for the duration of the sub-cycle. In addition there are special cases that must be accounted for. In the case $\alpha = 1$ and $y \neq 1$, $\theta_{(1,i)}(l,y) = 0$ since, if user 1 is backlogged at $t_e^{(r)}$, it will transmit in the first transmission period of the round with probability one. Similarly $\theta_{(0,i)}(l,1) = 0$, $i \neq 0$ since user 1 cannot transmit in the first transmission period if it is idle and other users are active at the beginning of the round. In the case where all users are idle at the beginning of the round ($S(t_e^{(r)}) = (0,0)$) then the channel remains idle until some user generates a packet. Transmissions then begin at the beginning of the next slot. In this case we can consider the round to be one with $S(t_e^{(r)}) = (\alpha, i)$ with probability $\frac{\binom{M-1}{i} p(X)^{i+1}[1-p(X)]^{M-i-1}}{1-[1-p(X)]^M}$ if $\alpha = 1$ or $\frac{\binom{M-1}{i} p(X)^{i}[1-p(X)]^{M-i}}{1-[1-p(X)]^M}$ if $\alpha = 0$. That is, the case $\alpha = i = 0$ is a convex combination of all the cases $\alpha + i \neq 0$. Thus $\theta_{(\alpha,i)}(l,y)$ is given by

$$\theta_{(\alpha,i)}(l,y) = \begin{cases} 0 & y=1,\ \alpha=0,\ i\neq 0 \\ & \text{or } y \neq 1,\ \alpha = 1,\ \forall i \\ [1-p(lX)]^{M-l}\binom{M-i-1}{l-i} f(l+1, l-i, l) & y=0, \alpha=0, i\neq 0 \\ [1-p(lX)]^{M-l}\binom{M-i-1}{l-i-1} f(l-y+1, l-i-1, l) & y=1,\ \alpha=1,\ \forall i \\ [1-p(lX)]^{M-l}\binom{M-i-1}{l-i-1}[1-p((y-2)X)]p(X) & y>1,\ \alpha=0,\ i\neq 0 \\ \quad \bullet f(l-y+1, l-i-1, l) & \\ \sum_{m=0}^{M-1} \frac{\binom{M-1}{m} p(X)^{m+1}[1-p(X)]^{M-m-1}}{1-[1-p(X)]^M}\theta_{(1,m)}(l,y) & \forall y,\ \alpha=0,\ i=0 \\ \quad + \sum_{m=1}^{M-1} \frac{\binom{M-1}{m} p(X)^{m}[1-p(X)]^{M-m}}{1-[1-p(X)]^M}\theta_{(0,m)}(l,y) & \end{cases}$$

$$\alpha + i \leq l \leq M$$

$$0 \leq y \leq l \tag{51}$$

**Most downstream user tagged:** Consider a sub-cycle of length $L = l$ where user $M$ transmits in transmission period $TP_t = y > 0$. That part of the sub-cycle up to and including transmission period $y$ is of length $L_1 = y$ and that part which is after transmission period $y$ is of length $L_2 = l - y$. With these definitions $\theta_{(\alpha,i)}(l, y)$ can be expressed as

$$\begin{aligned} \theta_{(\alpha,i)}(l, y) &= \Pr\{L_1 = y,\ L_2 = l - y \mid S(t_e^{(r)}) = (\alpha, i)\} \\ &= \Pr\{L_1 = y \mid S(t_e^{(r)}) = (\alpha, i)\} \Pr\{L_2 = l - y \mid L_1 = y,\ S(t_e^{(r)}) = (\alpha, i)\}. \end{aligned} \tag{52}$$

We note that since user $M$, when active and backlogged, defers its transmission to all other active backlogged users, it will transmit only when no other user is waiting to do so. Therefore we can compute $\Pr\{L_1 = y \mid S(t_e^{(r)}) = (\alpha, i)\}$ from $F(y - 1 - i, y - 1)$ where $F(m, s)$ is the recursive function given in equation (32), by considering only the $M - 1$ non-tagged users. Hence

$$\begin{aligned} &\Pr\{L_1 = y \mid S(t_e^{(r)}) = (\alpha, i)\} \\ &\qquad = [p((y-1)X)]^{1-\alpha}[1 - p((y-1)X)]^{M-y}\binom{M-i-1}{y-i-1}F(y-i-1, y-1) \end{aligned} \tag{53}$$

where the term $[p((y-1)X)]^{1-\alpha}$ is the probability that user $M$ is ready with a packet to transmit at the beginning of transmission period $y$ and the term $[1 - p((y-1)X)]^{M-y}$ is the probability that none of the $M - y$ other users generate any packets in the first $y - 1$ transmission periods in the round.

Obviously any user who transmits after user $M$ has had its turn must have generated its packet either during or after the transmission of user $M$. Thus by conditioning on the number of users that become backlogged during this transmission period we can use the same recursive function to compute $\Pr\{L_2 = L - y \mid L_1 = y,\ S(t_e^{(r)}) = (\alpha, i)\}$ as

$$\begin{aligned} &\Pr\{L_2 = l - y \mid L_1 = y,\ S(t_e^{(r)}) = (\alpha, i)\} = [1 - p((l - y + 1)X)]^{M-l} \\ &\qquad \binom{M-y}{l-y} \sum_{m=0}^{l-y} \binom{l-y}{m} p(X)^m [1 - p(X)]^{l-y-m} F(l - y - m, l - y) \end{aligned} \tag{54}$$

By substituting equations (53) and (54) into (52) we can compute $\theta_{(\alpha,i)}(l,y)$ as

$$\theta_{(\alpha,i)}(l,y) = \begin{cases} 0 & \begin{array}{l} 0 < y \le i,\ \forall\alpha,\ i > 0 \\ y = 0,\ \alpha = 1,\ \forall i \end{array} \\ [1-p(lX)]^{M-l}\binom{M-i-1}{l-i}F(l-i,l) & y = 0,\ \alpha = 0,\ i > 0 \\ [1-p(lX)]^{M-l}[1-p((y-1)X)]^{l-y}\binom{M-i-1}{y-i-1}\binom{M-y}{l-y}F(y-i-1,y-1) & \\ \quad \bullet\, [p((y-1)X)]^{1-\alpha} \sum_{m=0}^{l-y} \binom{l-y}{m} p(X)^m [1-p(X)]^{l-y-m} F(l-y-m,l-y) & \\ & y > i,\ \alpha + i \neq 0 \\ \sum_{i=0}^{M-1} \frac{\binom{M-1}{i} p(X)^{i+1}[1-p(X)]^{M-i-1}}{1-[1-p(X)]^M} \theta_{(1,i)}(l,y) & \\ \quad + \sum_{i=1}^{M-1} \frac{\binom{M-1}{i} p(X)^{i}[1-p(X)]^{M-i}}{1-[1-p(X)]^M} \theta_{(0,i)}(l,y) & \\ & \forall y,\ \alpha = 0,\ i = 0 \end{cases} \tag{55}$$

A recursive approach is also used to compute $\phi$. Note that, although $\phi_{(l,y)}(\gamma,k)$ depends on the length of the round and the transmission period in which the tagged user transmits, it does not depend on which user is tagged. Consider a round of length $L$ and the $s^{th}$ transmission period in this round where $1 \le s \le L$. Define the function $g(x,m,s)$ as the probability that out of $s$ users who transmitted in the previous $s$ transmission periods, $m$ of them, *excluding the tagged user*, generated new packets during these $s$ transmissions given that the tagged user transmitted in the $x^{th}$ transmission period in the round. Note that for this function, $s$ and $x$ are measured from the *beginning* of the round.

For any $s > 0$ there are $s$ dormant users and at most $s-1$ of them can have new packets in their buffer since the user who has just completed its transmission could not have generated a new packet instantaneously. If the tagged user has transmitted in transmission period $y$ where $y < s$ then at most $s-2$ of the non-tagged users could have new packets in each of their single buffers. Hence $g(x,m,s)$ must satisfy

$$g(x,m,s)=0 \qquad \text{if } m \geq s,\ \forall s \text{ or if } m \geq s-1,\ s>x \tag{56}$$

For $s=1$ the number of dormant users who are backlogged must be zero with probability one by the same argument. Thus

$$g(x,0,1)=1 \qquad \forall x. \tag{57}$$

For $s>1$ with $m<s$ or $m<s-1$, $s>x$, $g(x,m,s)$ can be computed recursively in terms of $g(x,m',s-1)$ by considering the new arrivals to dormant users in the transmission period $s$. We distinguish two cases: Either $s \leq x$ in which case there are $m-m'$ dormant users out of a possible $s-1-m'$ who generate new packets in transmission period $s$ or $s>x$, in which case there are $m-m'$ dormant users out of a possible $s-2-m'$ who generate new packets in transmission period $s$. This gives

$$g(x,m,s)=\begin{cases} \sum_{j=0}^{m}\binom{s-1-j}{m-j}p(1)^{m-j}[1-p(1)]^{s-1-m}g(x,j,s-1) & s \leq x \\ \sum_{j=0}^{m}\binom{s-2-j}{m-j}p(1)^{m-j}[1-p(1)]^{s-2-m}g(x,j,s-1) & s>x \end{cases} \tag{58}$$

$\phi_{(l,y)}(\gamma,k)$ is now computed from $g(x,m,s)$ and the probability of some arrivals during the inter-round gap

$$\phi_{(l,y)}(\gamma,k)=\begin{cases} \sum_{j=0}^{k} g(l+1,j,l)\binom{M-1-j}{k-j}[p(Y)]^{k-j+\gamma}[1-p(Y)]^{M-k-\gamma} & y=0 \\ [p((l-y)X+Y)]^{\gamma}[1-p((l-y)X+Y)]^{1-\gamma} & \\ \qquad \bullet \sum_{j=0}^{k} g(y,j,l)\binom{M-1-j}{k-j}[p(Y)]^{k-j}[1-p(Y)]^{M-1-k} & y \neq 0 \end{cases} \tag{59}$$

The elements of $\mathbf{P}$ can be computed directly from $\theta$ and $\phi$. From $\mathbf{P}$ we can calculate the stationary distribution $\Pi$.

In order to compute the distribution of delay for a packet from the tagged user we consider an arbitrary arrival to the system from this user in a given cycle. This arrival can be categorized into one of four types, each with a relatively simple distribution of delay. The Laplace transform of the distribution of delay of a packet of type $v$, $1 \leq v \leq 4$, which is generated in round $r$ where $S(t_e^{(r)}) = (\alpha, i)$, $S(t_e^{(r+1)}) = (\gamma, k)$, $L = l$ and $TP_t = y$, is denoted by $d^{*(v)}_{(\alpha,i)(\gamma,k)l,y}(s)$. This is derived for each packet type.

**Packets of type 1** are those that are generated by the tagged user during the first sub-cycle in a cycle where user 1 has not yet transmitted. Suppose that the tagged user transmits in transmission period $TP_t$. In the case where user 1 is the tagged user this packet must have been generated in transmission period $TP_t - 1$. In the case where user $M$ is the tagged user, this packet must have been generated at some time during the previous $TP_t - 1$ transmission periods. The distribution of delay of such a packet is given by

$$d^{*(1)}_{(\alpha,i)(\gamma,k)l,y}(s) = \begin{cases} \mathcal{E}^*(X, s)e^{-sX} & \text{user 1 tagged} \\ \mathcal{E}^*((y-1)X, s)e^{-sX} & \text{user } M \text{ tagged} \end{cases} \tag{60}$$

**Packets of type 2** are those that arrive during sub-cycle 2 (i.e., the inter-round gap). The delay incurred by this packet consists of two parts. The first part is the delay incurred until the beginning of the next cycle, the distribution of which is given by $\mathcal{E}^*(Y, s)$; the second part is the delay incurred in the next round while the tagged user waits for a turn to transmit. The distribution of this delay, conditioned on $S(t_e^{(r+1)}) = (1, k)$ is given by conditioning on $L$ and $TP_t$ in the $(r+1)^{st}$ round, as

$$\sum_{l'=k+1}^{M} \sum_{y'=1}^{l'} \theta_{(1,k)}(l', y')e^{-y'sX}. \tag{61}$$

Since the total delay of a packet of this type is the sum of two independent random variables, the distribution of delay in the Laplace domain is given by the product of the distributions of each part. Hence

$$d^{*(2)}_{(\alpha,i)(\gamma,k)l,y}(s) = \mathcal{E}^*(Y,s) \sum_{l'=k+1}^{M} \sum_{y'=1}^{l'} \theta_{(1,k)}(l',y')e^{-y'sX} \tag{62}$$

Note that for the case where user 1 is the tagged user, $\theta_{(1,k)}(l',y') = 0$ for $y' \neq 1$ and so equation (62) reduces to $d^{*(2)}_{(\alpha,i)(\gamma,k)l,y}(s) = \mathcal{E}^*(Y,s)e^{-sX}$.

**Packets of type 3** are the packets that are generated when the tagged user is dormant, that is, after the tagged user has already transmitted in the current round. Given that the packet is generated in round $r$ then the delay incurred in this round, conditioned on $L = l$ and $TP_t = y$ has a distribution given by $\mathcal{E}^*((l-y)X + Y, s)$ and the delay incurred in round $(r+1)$ conditioned on $n_u(t_e^{(r+1)}) = k$ has a distribution given by the expression in (61). Hence the distribution of delay for a packet of type 3, given by the product of these distributions, is

$$d^{*(3)}_{(\alpha,i)(\gamma,k)l,y}(s) = \mathcal{E}^*((l-y)X + Y, s) \sum_{l'=k+1}^{M} \sum_{y'=1}^{l'} \theta_{(1,k)}(l',y')e^{-y'sX}. \tag{63}$$

**Packets of type 4** distinguish the special case where the system is idle at the beginning of the round. Packets of this type are those that are generated by the tagged user in a round where $S(t_e^{(r)}) = (0,0)$ and are transmitted in the first sub-cycle of this cycle. In the case where user 1 is the tagged user, this packet will be transmitted in the slot immediately following the one in which it was generated and therefore is indistinguishable from a packet of type 1; however, in the case where user $M$ is tagged, the distribution of delay of this packet must take into account the idle slot immediately preceeding the first transmission of the round, in which the packet could have been generated. The distribution of delay of a packet of type 4 is

$$d^{*(4)}_{(\alpha,i)(\gamma,k)l,y}(s) = \begin{cases} \mathcal{E}^*(X,s)e^{-sX} & \text{user 1 tagged} \\ \mathcal{E}^*(yX,s)e^{-sX} & \text{user } M \text{ tagged} \end{cases} \tag{64}$$

Let $\varsigma^{(v)}_{(\alpha,i),(\gamma,k),l,y}$ denote the probability that an arbitrary arrival from the tagged user is in a cycle with $S(t_e^{(r)}) = (\alpha, i)$, $S(t_e^{(r+1)}) = (\gamma, k)$, $L = l$, $TP_t = y$ and is of type $v$. $\varsigma^{(v)}_{(\alpha,i),(\gamma,k),l,y}$ is also the fraction of packets which are of type $v$ and arrive in such a round. We now show how to derive $\varsigma$ and then use this to compute the distribution of delay for a packet from the tagged user by removing the conditions on the expressions given in equations (60) through (64) for the distribution of delay.

To compute $\varsigma^{(v)}_{(\alpha,i),(\gamma,k),l,y}$ we first derive $\Pr\{S(t_e^{(r)}) = (\alpha, i),\ S(t_e^{(r+1)}) = (\gamma, k),\ L = l,\ TP_t = y\}$ as follows

$$
\begin{aligned}
&\Pr\{S(t_e^{(r)}) = (\alpha, i),\ S(t_e^{(r+1)}) = (\gamma, k),\ L = l,\ TP_t = y\} \\
&\quad = \Pr\{S(t_e^{(r)}) = (\alpha, i)\}\Pr\{S(t_e^{(r+1)}) = (\gamma, k),\ L = l,\ TP_t = y \mid S(t_e^{(r)}) = (\alpha, i)\} \\
&\quad = \Pr\{S(t_e^{(r)}) = (\alpha, i)\}\Pr\{L = l,\ TP_t = y \mid S(t_e^{(r)}) = (\alpha, i)\} \\
&\qquad\qquad \bullet \Pr\{S(t_e^{(r+1)}) = (\gamma, k) \mid L = l,\ TP_t = y\} \\
&\quad = \pi_{(\alpha,i)}\theta_{(\alpha,i)}(l, y)\phi_{(l,y)}(\gamma, k)
\end{aligned}
\tag{65}
$$

In an arbitrary cycle, a packet of type $v$, $1 \leq v \leq 4$ is generated by the tagged user if and only if this cycle meets certain conditions. A packet of type one is generated if $\delta(t_e^{(r)}) = 0$, $n(t_e^{(r)}) > 0$ and $TP_t > 0$. A packet of type 2 is generated if $\delta(t_e^{(r)}) = 0$, $n(t_e^{(r)}) > 0$, $\delta(t_e^{(r+1)}) = 1$ and $TP_t = 0$. A packet of type 3 is generated if $TP_t > 0$ and $\delta(t_e^{(r+1)}) = 1$. A packet of type 4 is generated if $S(t_e^{(r)}) = (0, 0)$ and $TP_t > 0$. For any other conditions no packets are generated by the tagged user. Consider now a large number of cycles with $S(t_e^{(r)}) = (\alpha, i)$, $S(t_e^{(r+1)}) = (\gamma, k)$, $L = l$, $TP_t = y$. Suppose that there are $Z$ such cycles. In the limit as $Z \to \infty$, the number of packets of type $v$ is given by $Z$ times the probability that the tagged user generates a packet of type $v$ in a cycle. This probability is given by

$$\Pr\{\text{Tagged user generates a packet of type } v\} = \begin{cases} \pi_{(0,i)}\theta_{(0,i)}(l,y)\phi_{(l,y)}(\gamma,k) & v=1, i>0, y>0 \\ \pi_{(0,i)}\theta_{(0,i)}(l,0)\phi_{(l,0)}(1,k) & v=2, i>0 \\ \pi_{(\alpha,i)}\theta_{(\alpha,i)}(l,y)\phi_{(l,y)}(1,k) & v=3, y>0, \\ \pi_{(0,0)}\theta_{(0,0)}(l,y)\phi_{(l,y)}(\gamma,k) & v=4, y>0 \end{cases} \tag{66}$$

The probability that an arbitrary arrival is of type $v$ is given by the ratio of the number of packets generated by the tagged user which are of type $v$ divided by the total number of packets generated by the tagged user. Hence $\varsigma$ is given by

$$\varsigma^{(v)}_{(\alpha,i)(\gamma,k),l,y} = \begin{cases} K\pi_{(0,i)}\theta_{(0,i)}(l,y)\phi_{(l,y)}(\gamma,k) & v=1, \alpha=0, i>0, y>0 \\ K\pi_{(0,i)}\theta_{(0,i)}(l,0)\phi_{(l,0)}(1,k) & v=2, \alpha=0, \gamma=1, i>0, y=0 \\ K\pi_{(\alpha,i)}\theta_{(\alpha,i)}(l\ y)\phi_{(l,y)}(1,k) & v=3, y>0, \gamma=1 \\ K\pi_{(0,0)}\theta_{(0,0)}(l,y)\phi_{(l,y)}(\gamma,k) & v=4, \alpha=0, i=0, y>0 \\ 0 & \text{otherwise} \end{cases} \tag{67}$$

where $K$ is a constant such that

$$\sum_{v=1}^{4}\sum_{\alpha=0}^{1}\sum_{i=0}^{M-1}\sum_{\gamma=0}^{1}\sum_{k=0}^{M-1}\sum_{l=\max(1,\alpha+i)}^{M}\sum_{y=0}^{l}\varsigma^{(v)}_{(\alpha,i)(\gamma,k),l,y} = 1 \tag{68}$$

From $\varsigma$ and the equations for the delay given in (60) through (64), it is straight forward to compute the Laplace transform of the distribution of delay for a packet generated by the tagged user. Denoting this distribution by $D^*(s)$ we get

$$D^*(s) = \sum_{v=1}^{4}\sum_{\alpha=0}^{1}\sum_{i=0}^{M-1}\sum_{\gamma=0}^{1}\sum_{k=0}^{M-1}\sum_{l=\alpha+i}^{M}\sum_{y=0}^{l}\varsigma^{(v)}_{(\alpha,i)(\gamma,k),l,y}d^{*(v)}_{(\alpha,i)(\gamma,k)l,y}(s). \tag{69}$$

By successive differentiation of (69) and letting $s=0$, one can compute the moments of delay to any order.

## 5. Numerical Results

We discuss in this section numerical results showing the performance of Express-net show Express-net operating under the NGSS discipline and Fasnet operating under the GSS and MUFS disciplines, we also present some representative figures showing Express-net operating under GSS and Fasnet operating under NGSS. Recall that only Fasnet can support the MUFS service discipline. Let $a \triangleq \tau/T$. The unit of time is taken to be the transmission time of a packet (i.e., $T = 1$). In both Express-net and Fasnet we neglect the inter-packet overhead $t_o$ since this is assumed to be small compared to the length of the packet. The inter-round overhead $Y$ is then taken to be $2a$ for Express-net and $\lceil 2a \rceil + 1$ for Fasnet. The performance of these networks for various values of $a$ and $M$ is presented in terms of the throughput as a function of the generation rate of packets, the maximum channel utilization referred to as the network capacity, and the throughput-delay trade-off. These results show that all three service disciplines exhibit similar performance characteristics. This is to be expected since they are merely variations of a basic round robin algorithm. However there are interesting differences which we will highlight in the discussion. All numerical results are obtained from analysis with the exception of MUFS when $Y > 1$ in which case simulation is used. The reason is that, as pointed out in section 4, the analysis for MUFS gives pessimistic results when $Y > 1$. In most of the results shown below, the preamble in Express-net has been assumed to be negligible except for certain figures where its effect is explicitly shown.

In Figs. 6, 7 and 8 we show the behavior of the throughput $S$ as a function of the aggregate generation rate $M\lambda$ for $a = 1.0$ and 10, and $M = 20$, 30 and 50, for NGSS in Express-net, and GSS and MUFS in Fasnet. $M\lambda$ is the rate at which packets would be presented to the system if all users were in the idle state. The curves show that $S$ increases steadily as $M\lambda$ increases from zero until some finite value of $M\lambda$ (in the vicinity of one), and remains practically constant as $M\lambda$ increases further. This shows that the system remains stable as the load increases to infinity. (Contrast this to CSMA-CD where stability can only be achieved by using some form of dynamic control or a long rescheduling

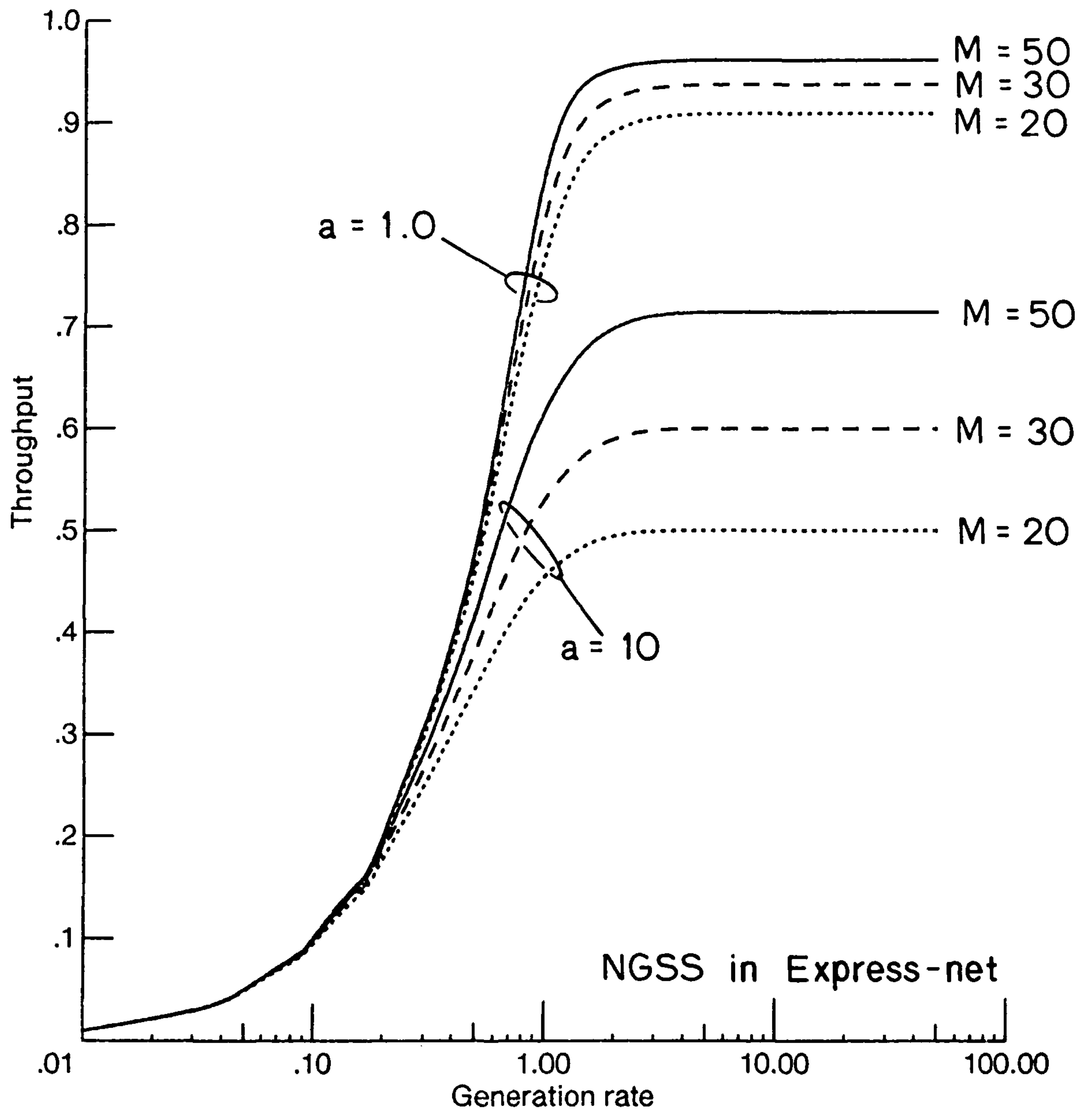


Fig. 6 Channel throughput as a function of the generation rate $M\lambda T$ for NGSS operating in Express-net.

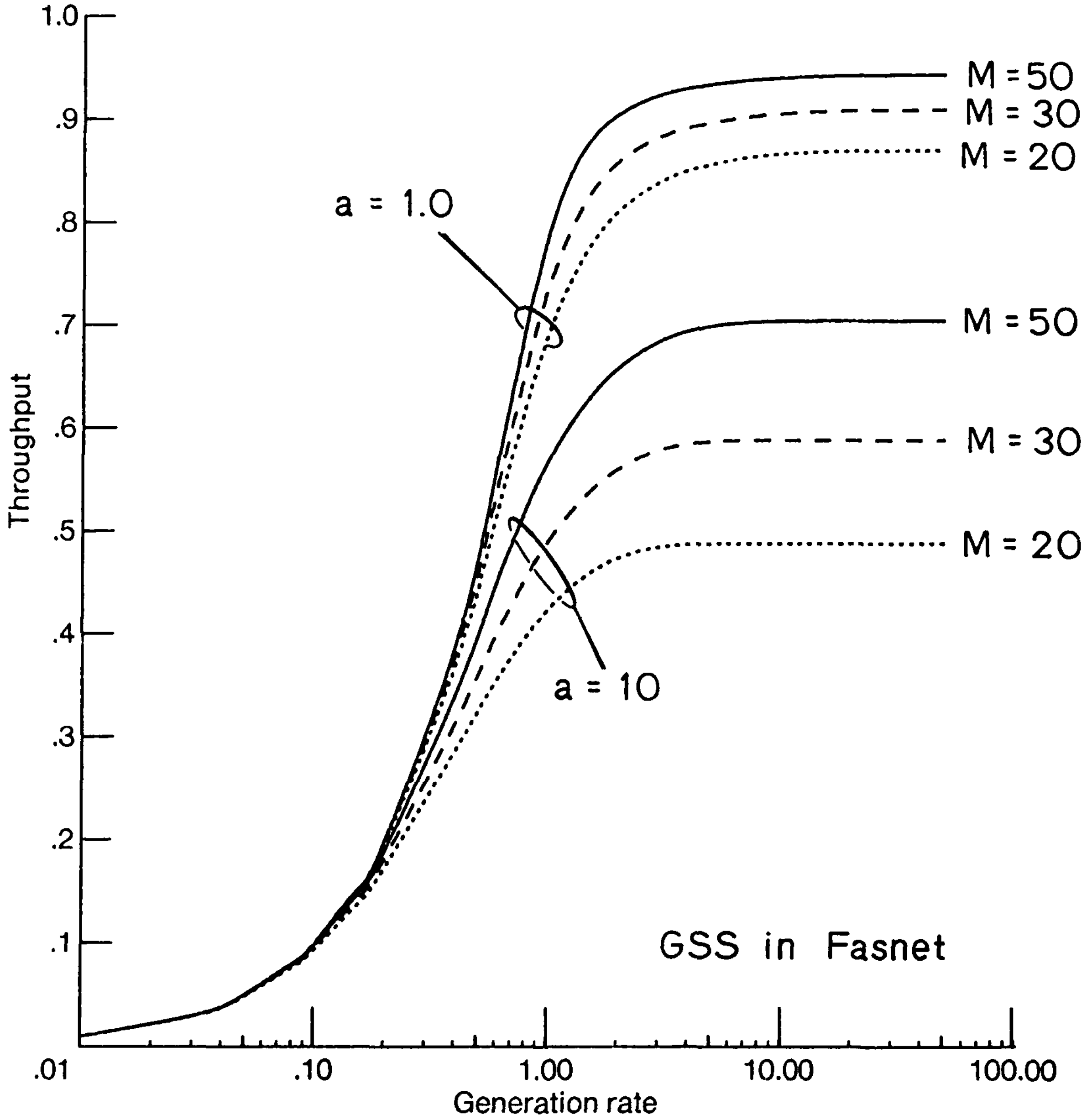


Fig. 7 Channel throughput as a function of the generation rate $M\lambda T$ for GSS operating in Fasnet.

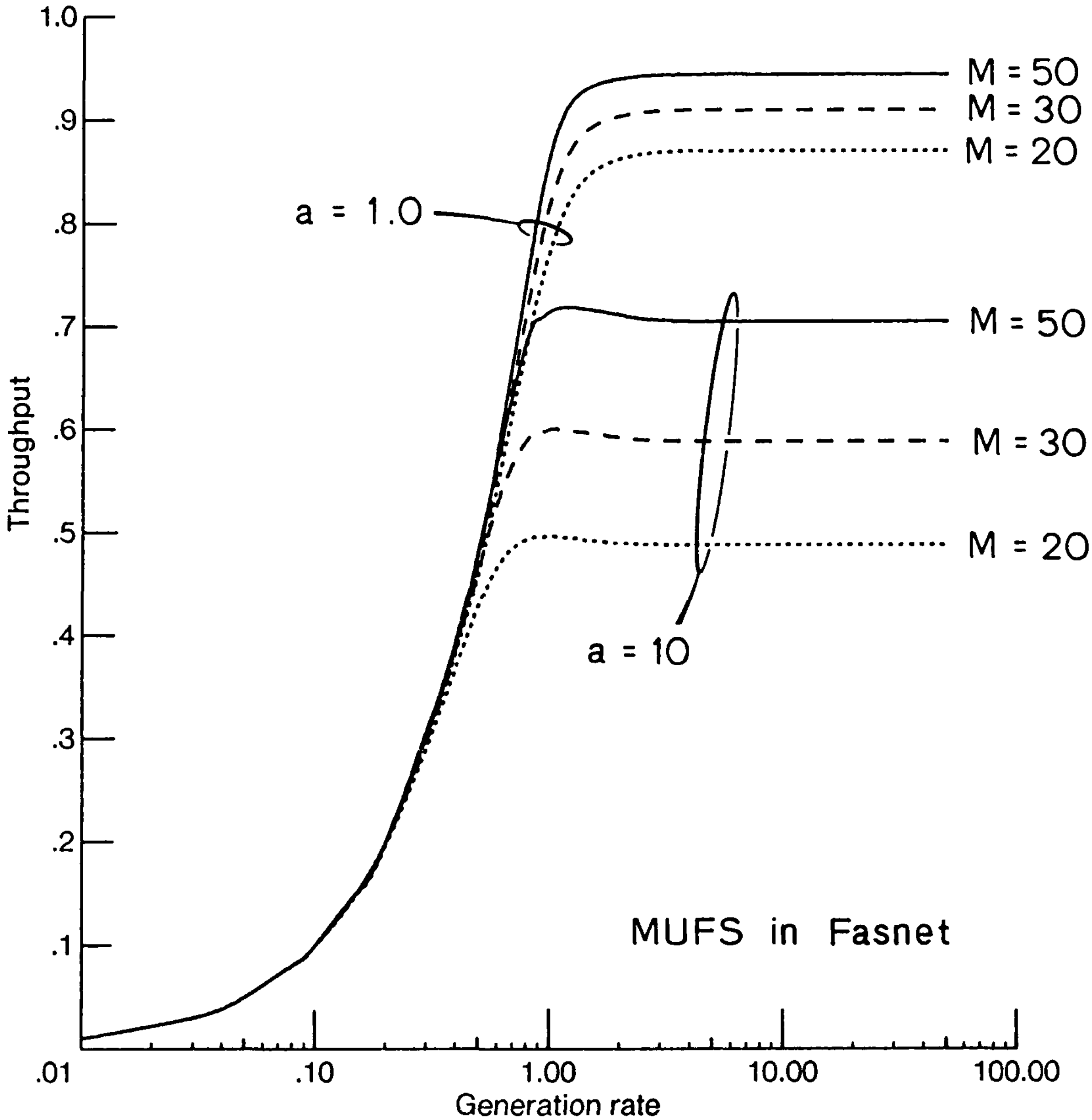


Fig. 8 Channel throughput as a function of the generation rate $M\lambda T$ for MUFS operating in Fasnet. The curves for $a = 10$ were obtained by simulation.

obtained by simulation,) exhibit a slight hump before $S$ levels off to its constant value. This occurs since, at the generation rate corresponding to the hump in $S$, all users are on the average transmitting in every round, but some users happen to generate and transmit their packets during the inter-round overhead; this results in a *lower* effective overhead and hence higher throughput than expected. As mentioned in Section 2, it is possible to operate Express-net under the GSS discipline and Fasnet under the NGSS discipline.

In order to show the effect of the service discipline on the channel throughput, we plot in Fig. 9 for Express-net and in Fig. 10 for Fasnet $S$ vs. $M\lambda$ for each of these two service disciplines. Note how, as a result of gating (i.e., the delaying of packets until the round following the one in which they were generated), the throughput achieved by GSS is always less than or equal to that achieved by NGSS.

In Fig. 11 we show on a single sheet for comparison purposes, $S$ vs. $M\lambda$ for the three service disciplines. As the network reaches saturation, $S$ approaches a finite value, given by $\frac{M}{M(1+t_p)+Y}$ independent of the service discipline, which we call the network capacity. For NGSS and GSS the network capacity represents the maximum channel utilization. For MUFS, the maximum channel utilization is slightly higher than the network capacity for the reasons discussed above. The difference between the network capacity for NGSS and that for GSS and MUFS seen in Fig. 11 is a result of the different values of $Y$ in Express-net and Fasnet for the same value of $a$ ($2a$ and $\lceil 2a \rceil + 1$ respectively); recall that the preamble $t_p$ is assumed here to be zero. In Fig. 12 we plot for Express-net and Fasnet the network capacity versus $a$ for various values of $M$. Unlike CSMA-CD (see Fig. 1), a high utilization can still be achieved for large $a$ when $M$ is large. If $M$ is not sufficiently large then one can alter the access protocol to allow each user to transmit more than one packet in a round thereby achieving a high utilization as for large $M$. Due to the fixed slot size in Fasnet, the inter-round overhead does not decrease below two slots even as $a$ becomes very small; hence the poor channel utilization in the case of Fasnet for small $a$ when $M = 1$.

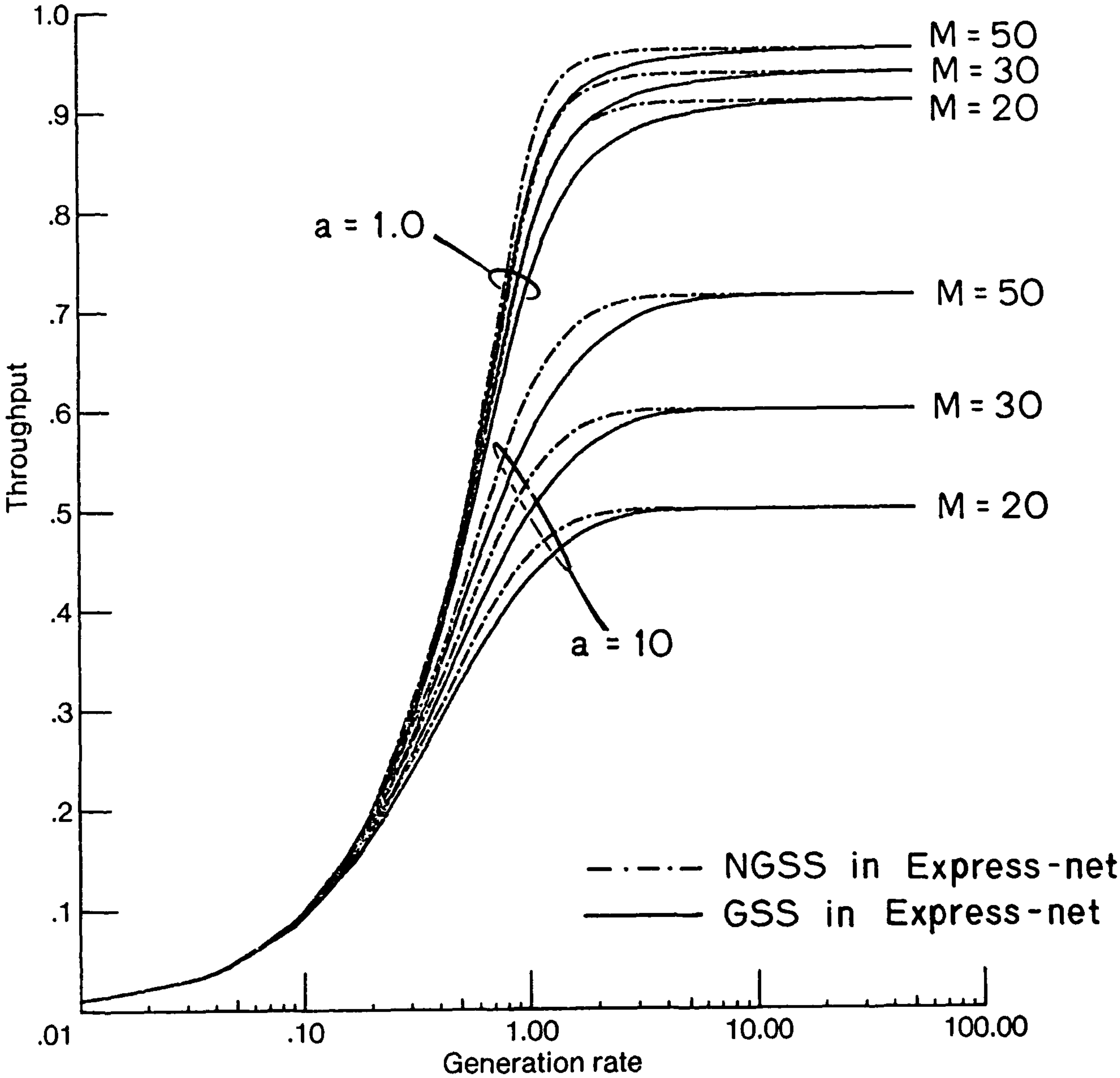


Fig. 9 Channel throughput as a function of the generation rate $M\lambda T$ showing the difference between NGSS and GSS operating in Express-net.

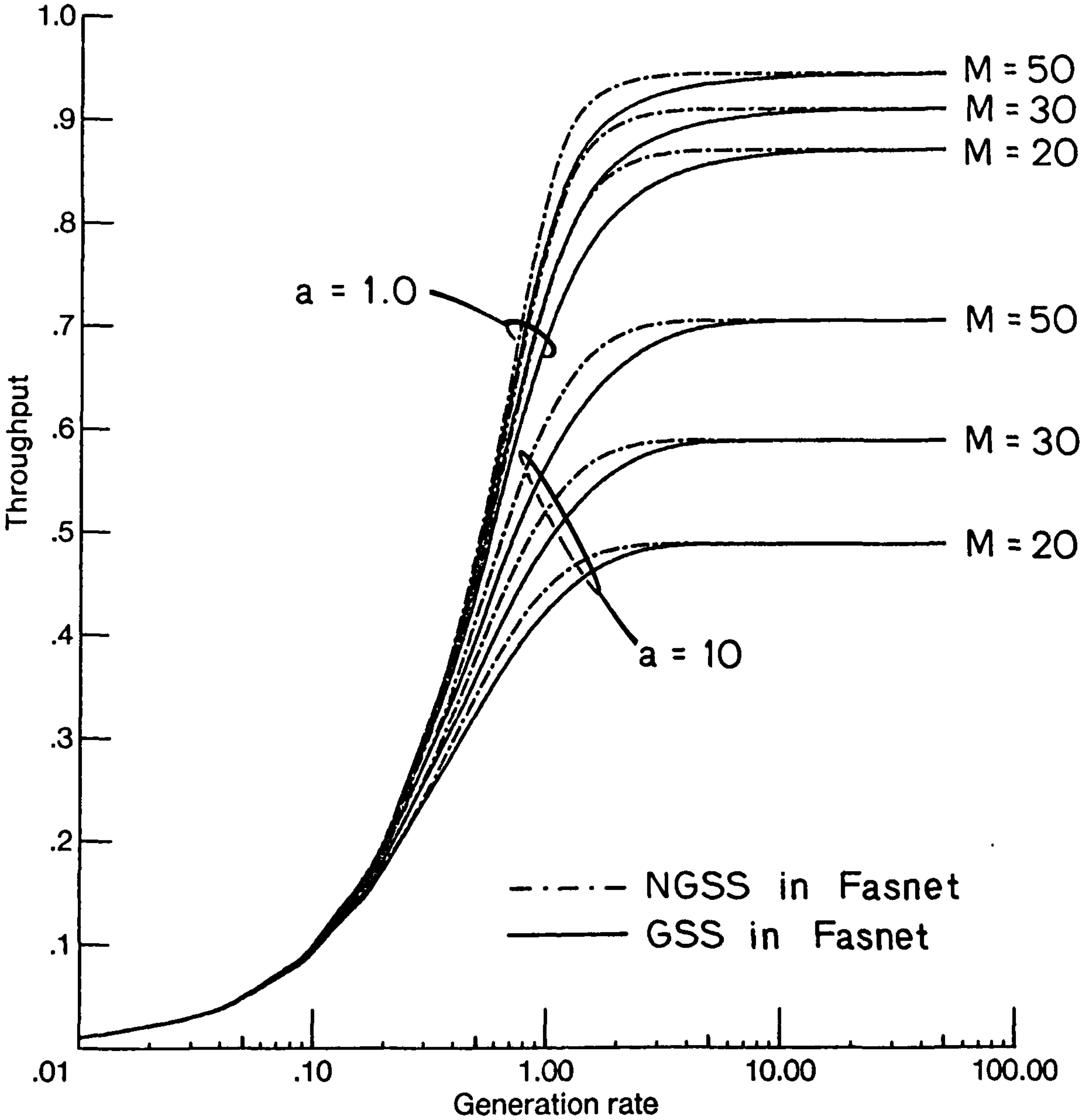


Fig. 10 Channel throughput as a function of the generation rate $M\lambda T$ showing the difference between NGSS and GSS operating in Fasnet.

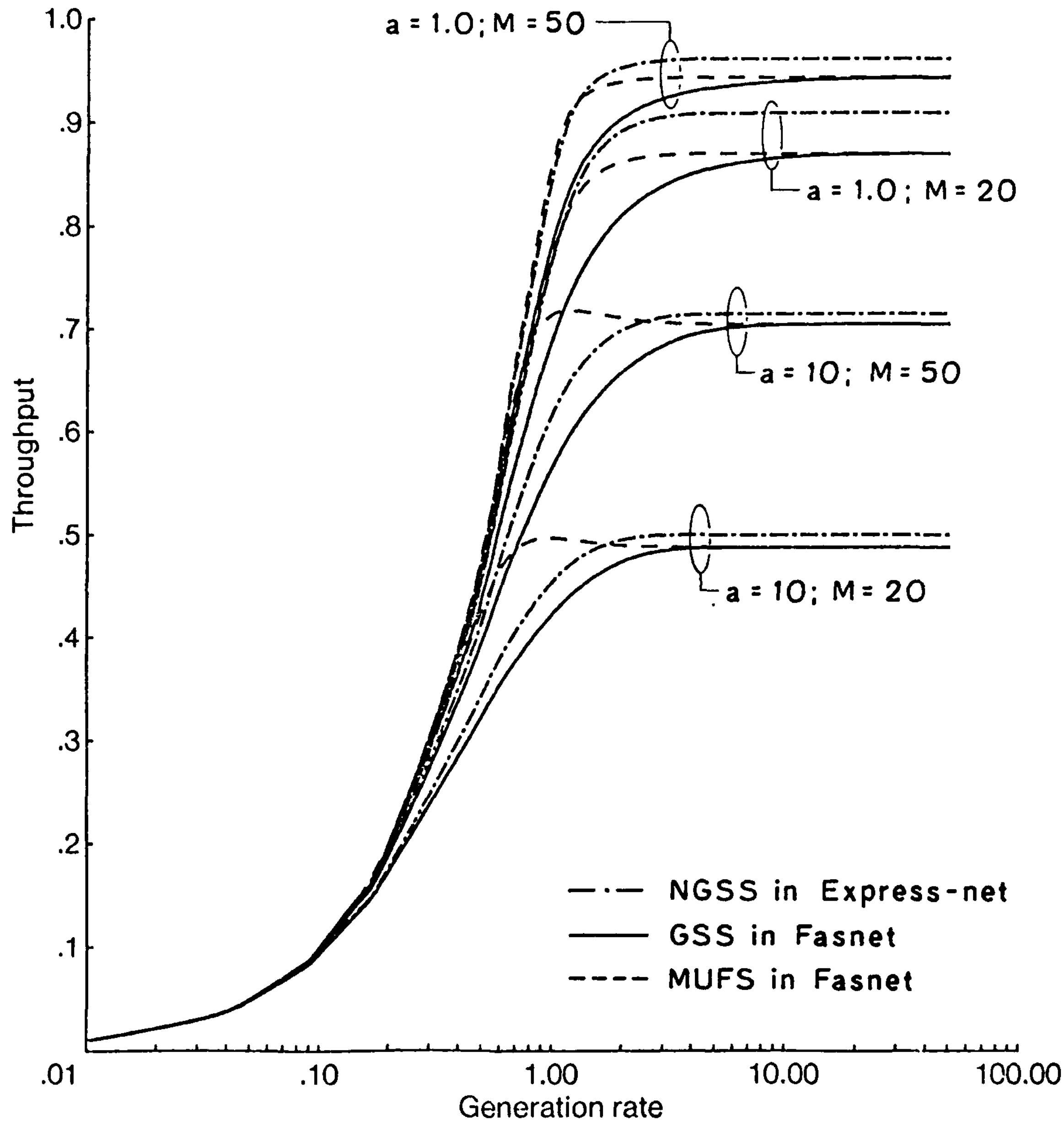


Fig. 11 Channel throughput as a function of the generation rate $M\lambda T$ comparing NGSS in Express-net, GSS in Fasnet and MUFS in Fasnet. The curves showing MUFS with $a = 10$ were obtained by simulation.

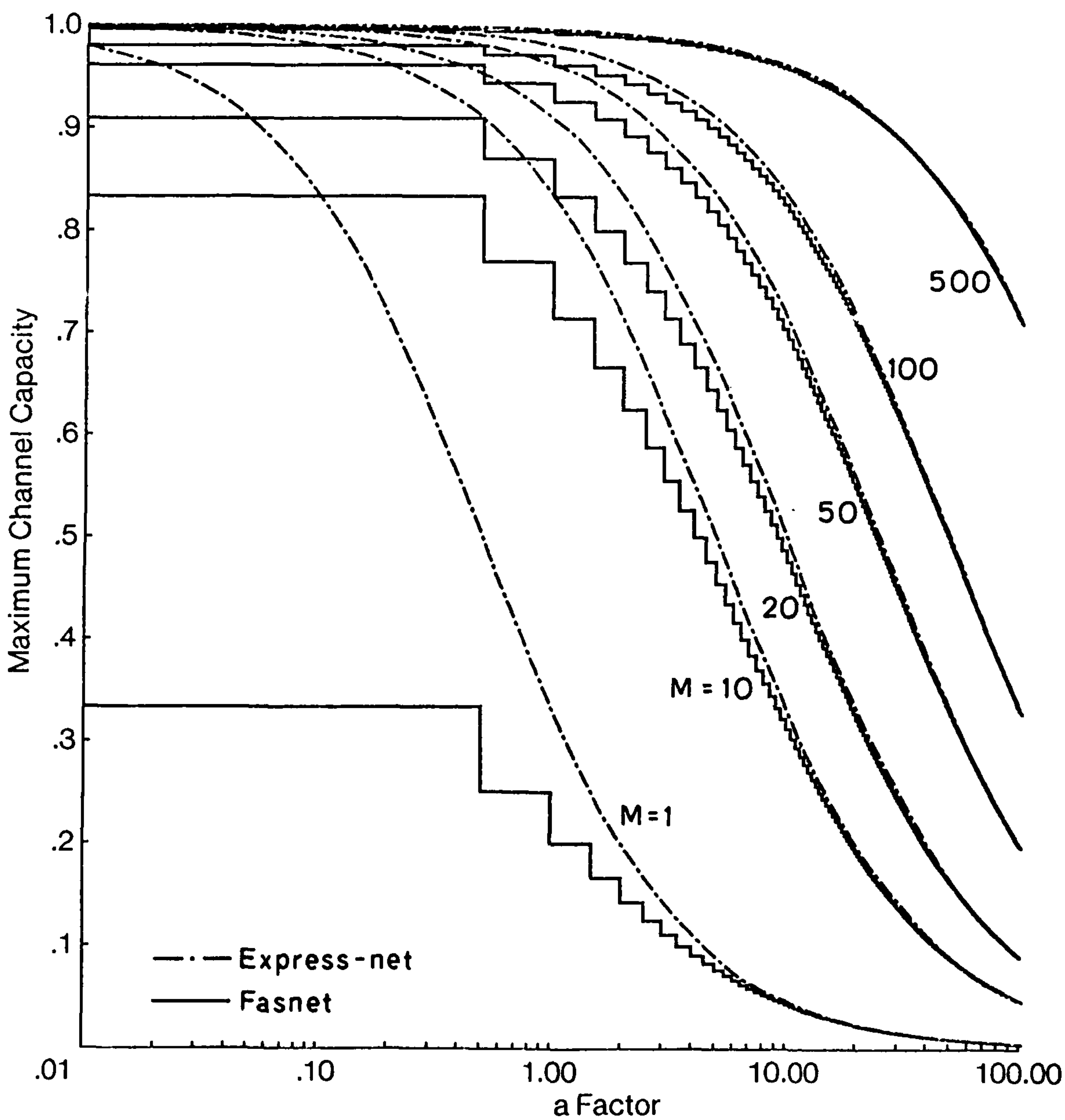


Fig. 12 Network capacity versus $a$ for Express-net and Fasnet.

In Express-net there is no slotting of the time axis and so as $a \rightarrow 0$ the overhead becomes zero and the maximum channel utilization goes to 1. The effect of a non-zero preamble on the network capacity for Express-net can be seen in Fig. 13 where some representative curves are plotted. A preamble which is on the same order of magnitude as the packet transmission time will cause a significant degradation in the capacity.

The relationship between $S$ and average delay $D$ normalized to $T$ for $a = 0.1$, 1.0 and 10, and $M = 20$, 30 and 50 is shown in Fig. 14 for NGSS in Express-net, Fig. 15 for GSS in Fasnet, and in Fig. 16 for MUFS in Fasnet. We see that, for a given $S$, $D$ is fairly insensitive to $M$ as long as $M$ is large enough so that this value of $S$ can be achieved. We also see that the *normalized* average delay increases as $a$ gets larger. However if $a$ $(= \tau W/B)$ has become larger because the channel bandwidth $W$ has increased or the packet size $B$ has decreased, meaning that $T$ $(= B/W)$ has decreased, then the *actual* delay is smaller than that obtained with small $a$; the packet transmission time has decreased thus reducing the size of a slot and the length of a round.* On the other hand, if $a$ has become larger because the size of the network has increased, meaning that $T$ has not changed, then the actual delay will have increased as represented by the normalized delay. Although the performance trends for all three disciplines are similar, the results do show some differences. These are highlighted in Figs. 17 and 18. We plot on a single sheet the throughput-delay trade-off for each of the three disciplines for $a = 0.1$ in Fig. 17, and for $a = 1.0$ and 10 in Fig. 18. In particular one should note that, for large $a$ $(a = 10)$, MUFS achieves substantially lower delay than the other two schemes as long as the throughput is not close to saturation. This is due to the fact that in MUFS, having generated a packet, a non-dormant user transmits this packet in the next available slot regardless of when the

*This can be exemplified with a special case. Consider a saturated system $(\lambda \rightarrow \infty)$ with $M$ users and inter-round overhead $Y$. Suppose that, for a given bandwidth, the transmission time is $T$. The overhead $t_o + t_p$ is assumed to be negligible. In this case the actual delay is $D = MT + Y$ and the normalized delay $(D/T) = M + Y/T$. Suppose now that the bandwidth of the system is increased meaning that $a$ increases and $T$ decreases. Let this new value of the transmission time be $T'$ where $T' < T$. In this case the normalized packet delay $(D'/T')$ is given by $(D'/T') = M + Y/T' > (D/T)$; i.e., the normalized delay has *increased*. The actual delay, however, is $D' = MT' + Y < D$; i.e., the actual delay has ***decreased***.

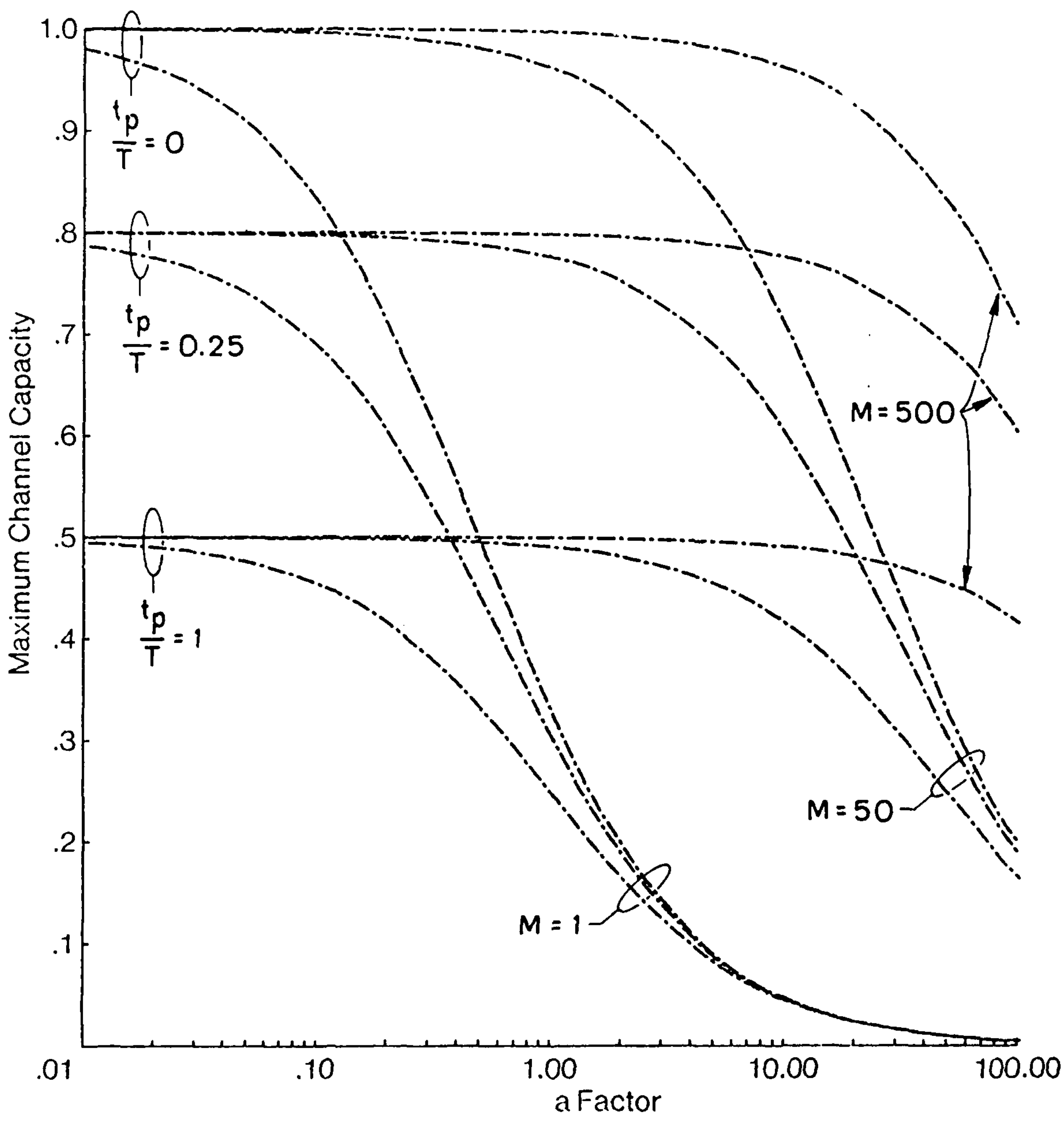


Fig. 13 Network capacity versus $a$ for Express-net with $M = 1$, 50 and 500, and with three values of the preamble corresponding to $t_p = 0$, $0.25T$ and $T$.

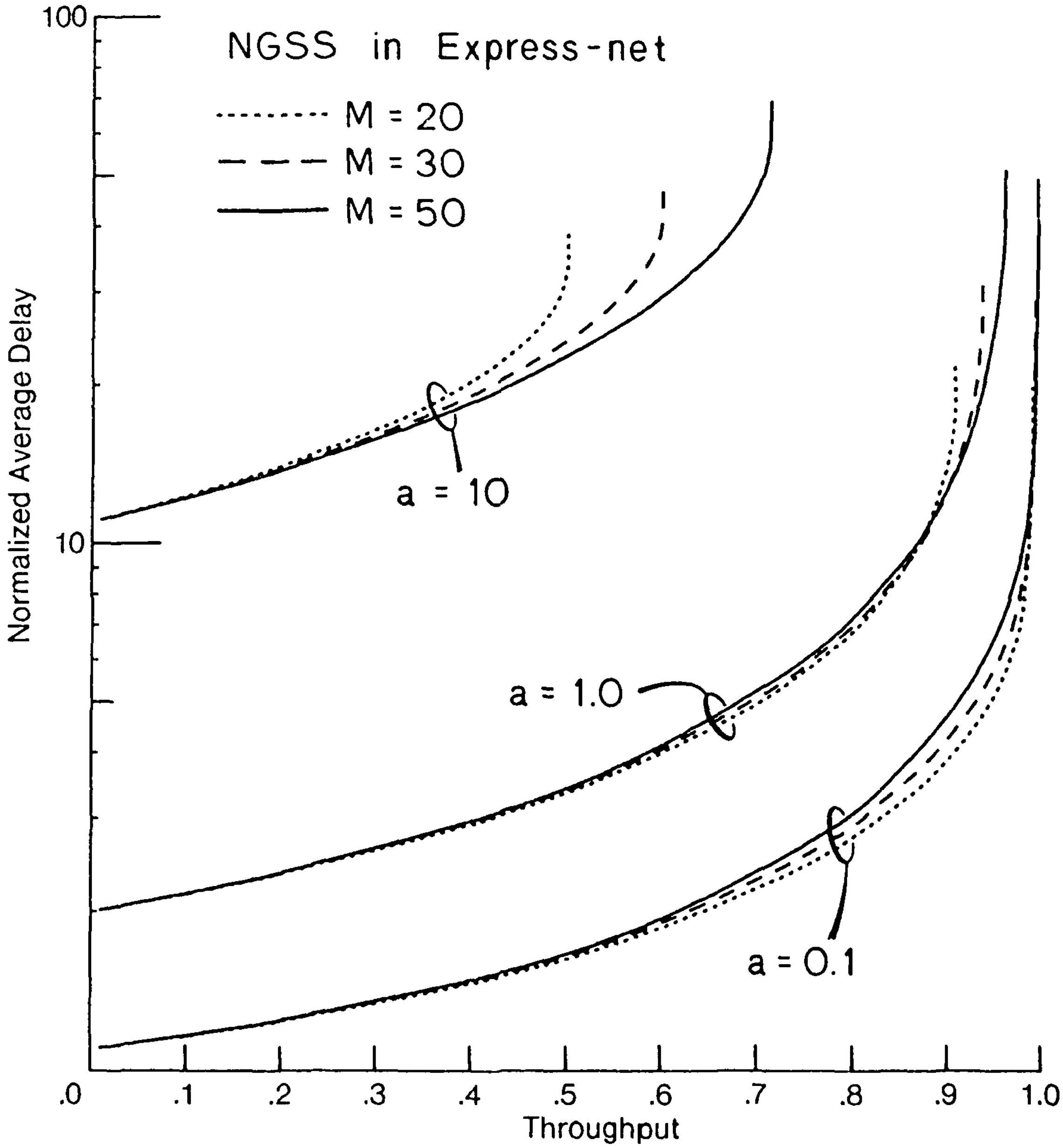


Fig. 14 Average delay normalized by the packet transmission time $T$ versus the channel throughput for NGSS in Express-net.

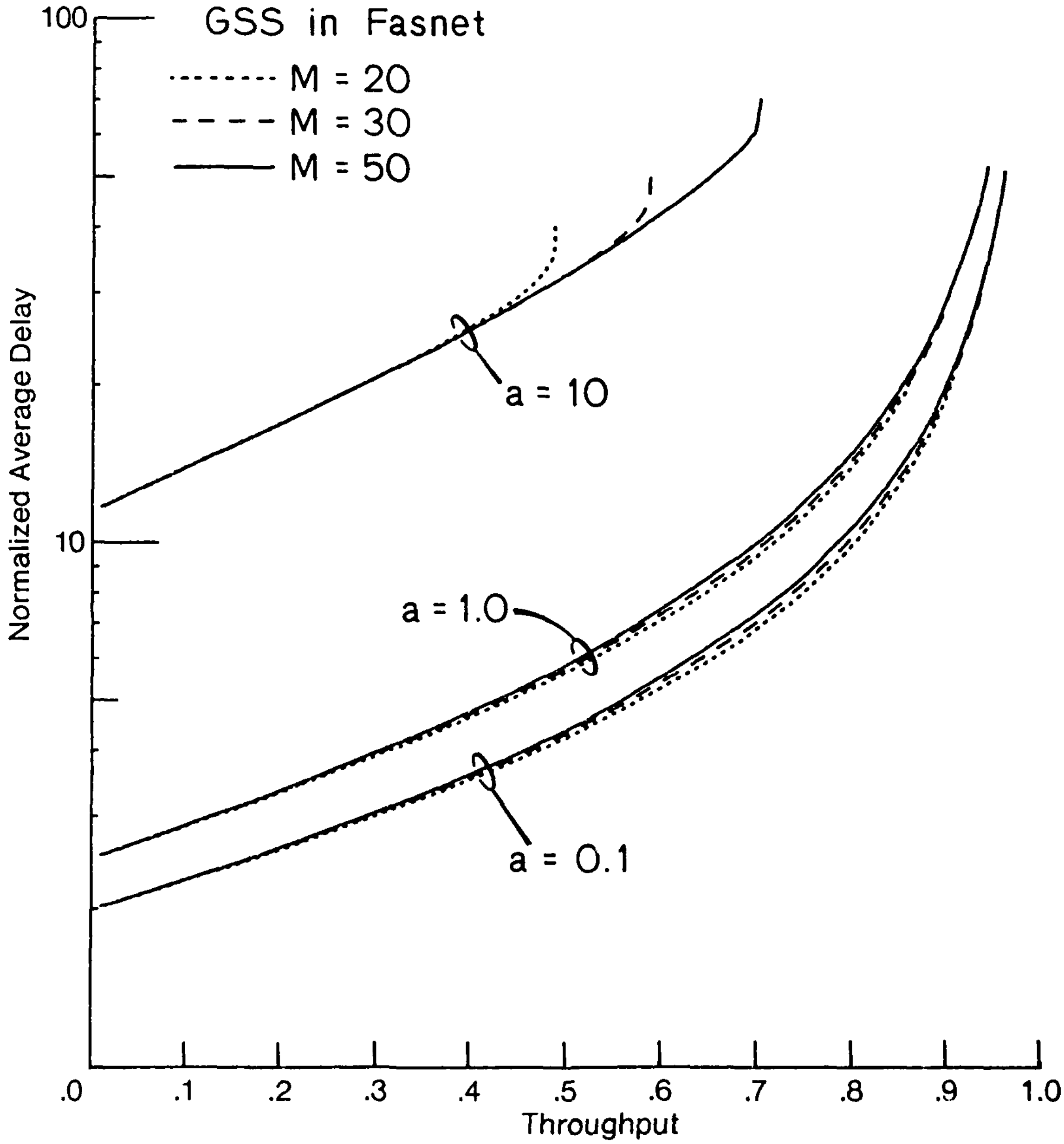


Fig. 15 Average delay normalized by the packet transmission time $T$ versus the channel throughput for GSS in Fasnet.

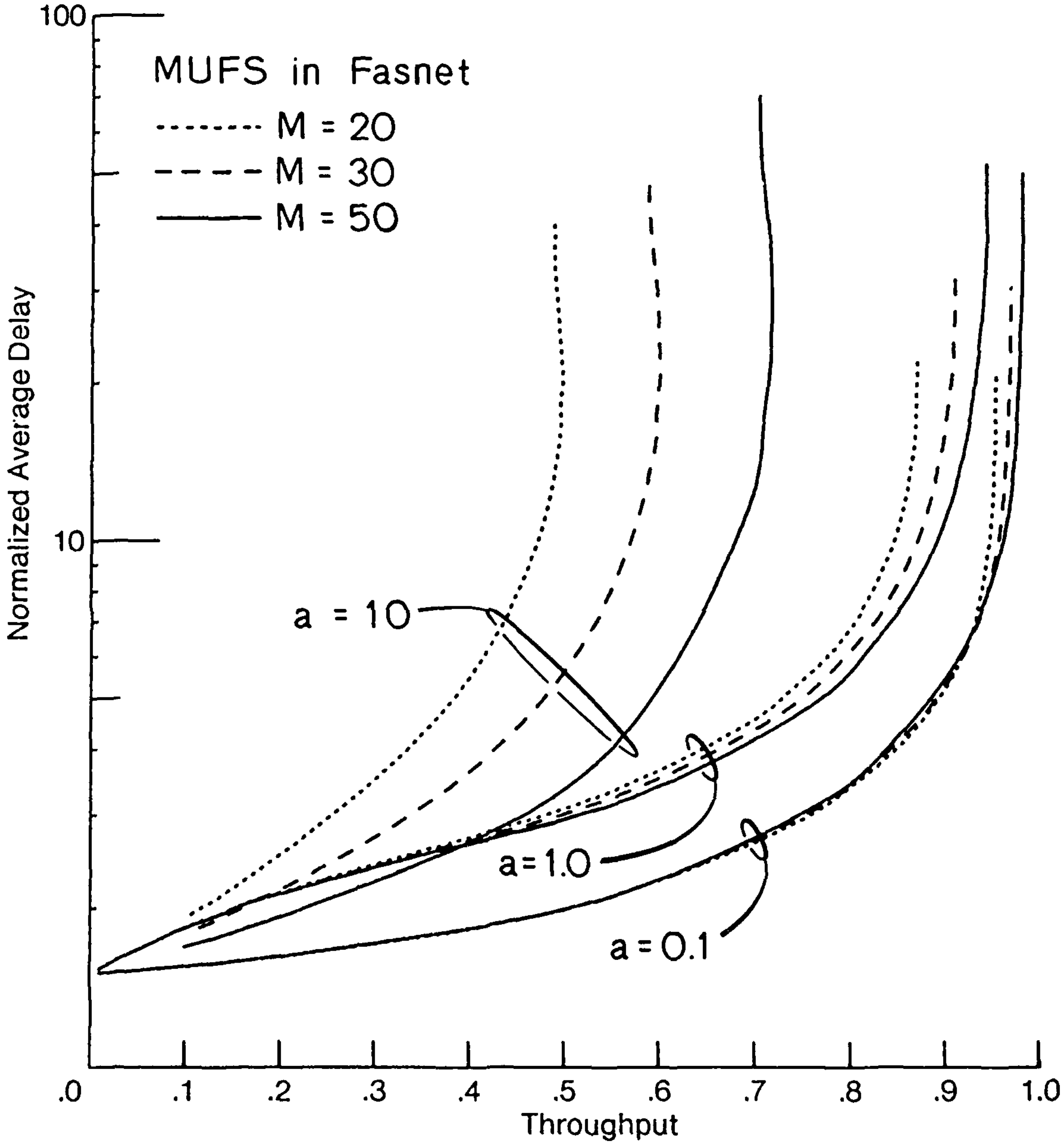


Fig. 16 Average delay normalized by the packet transmission time $T$ versus the channel throughput for MUFS in Fasnet.

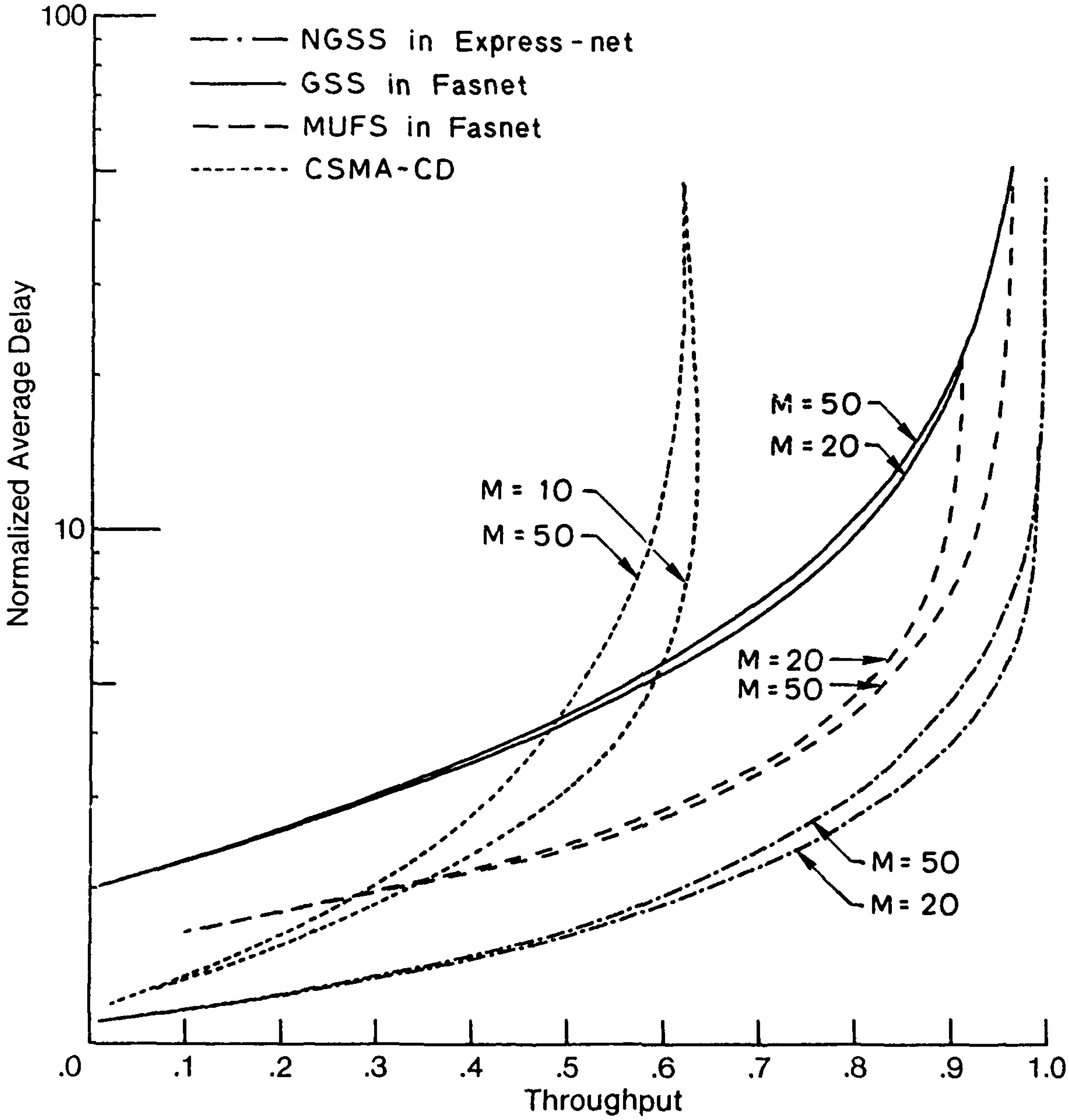


Fig. 17 Average delay normalized by the packet transmission time $T$ versus the channel throughput with $a = 0.1$, comparing NGSS, GSS, MUFS and CSMA-CD.

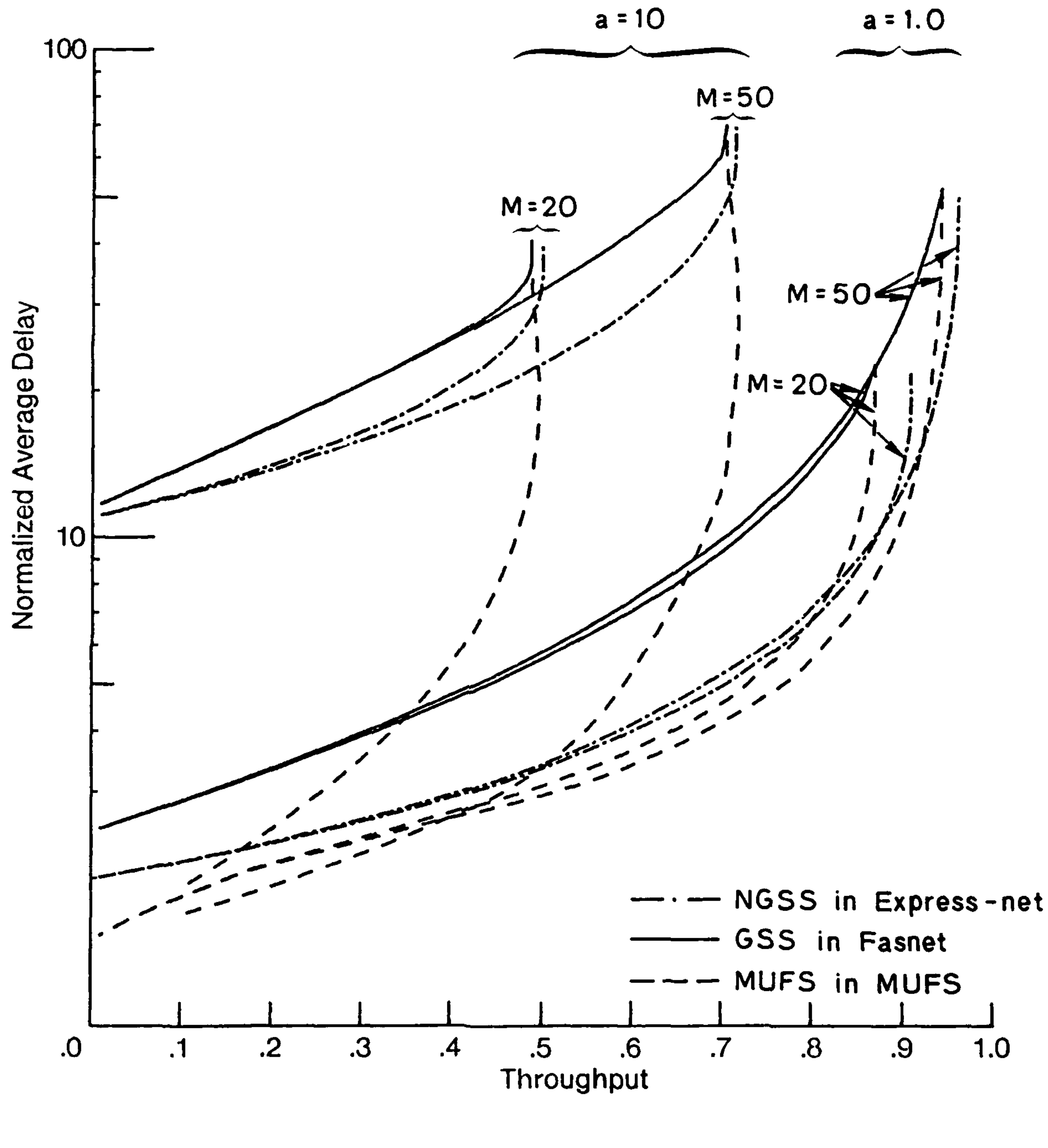


Fig. 18 Average delay normalized by the packet transmission time $T$ versus the channel throughput with $a = 1.0$ and 10 comparing NGSS, GSS and MUFS. The curves for MUFS with $a = 10$ were obtained by simulation. All the other curves were obtained from the analysis.

start of cycle appears. In particular, at $S = 0$, $D$ will be equal to $1.5T$ since a user can transmit its packet in the slot immediately following the one in which it was generated, instead of incurring on the average a delay of $aT$ while waiting for the SOC as in GSS, or the locomotive as in NGSS. [If $a$ is made large because $W$ is increased then $T$ decreases and so, in absolute terms, the delay $aT$ remains constant and equal to $\tau$.] Also in Fig. 17 is plotted the relationship between $S$ and $D$ for CSMA-CD.* As with Express-net, we assume that the preamble for CSMA-CD is negligible. This figure shows how favorably the delay performance of the round robin schemes compares to that of CSMA-CD. No throughput-delay curves are plotted for CSMA in Fig. 18 since for $a = 1.0$ and 10 this access scheme achieves a very small network capacity.

Operating Express-net in GSS mode results in a throughput-delay trade-off similar to that of GSS in Fasnet. However, for GSS in Express-net, there is a small improvement in the throughput-delay characteristics as compared to GSS in Fasnet, since, in Express-net, the inter-round overhead is smaller than that of Fasnet for a given $a$. We show in Fig. 19 the average packet delay as a function of the throughput for both Express-net and Fasnet operating under the GSS discipline. On the other hand, one may operate Fasnet under the NGSS discipline and achieve a throughput-delay trade-off similar to that shown in Fig. 14 for NGSS in Express-net. In this case however, there is a slight degradation in the throughput-delay characteristics as compared with NGSS in Express-net as shown in Fig. 20 where we plot the average packet delay as a function of the channel throughput for both Express-net and Fasnet operating under the NGSS discipline.

The average packet delay (normalized to $T$) versus $S$ for NGSS in Express-net, in the case where the preamble is not negligible, is plotted in Fig. 21 for $a = 1$ and 10 and $M = 50$. As expected, the delay for a given $S$ increases as the length of the preamble increases.

*The CSMA-CD scheme considered here is the slotted non-persistent version analysed in [2].

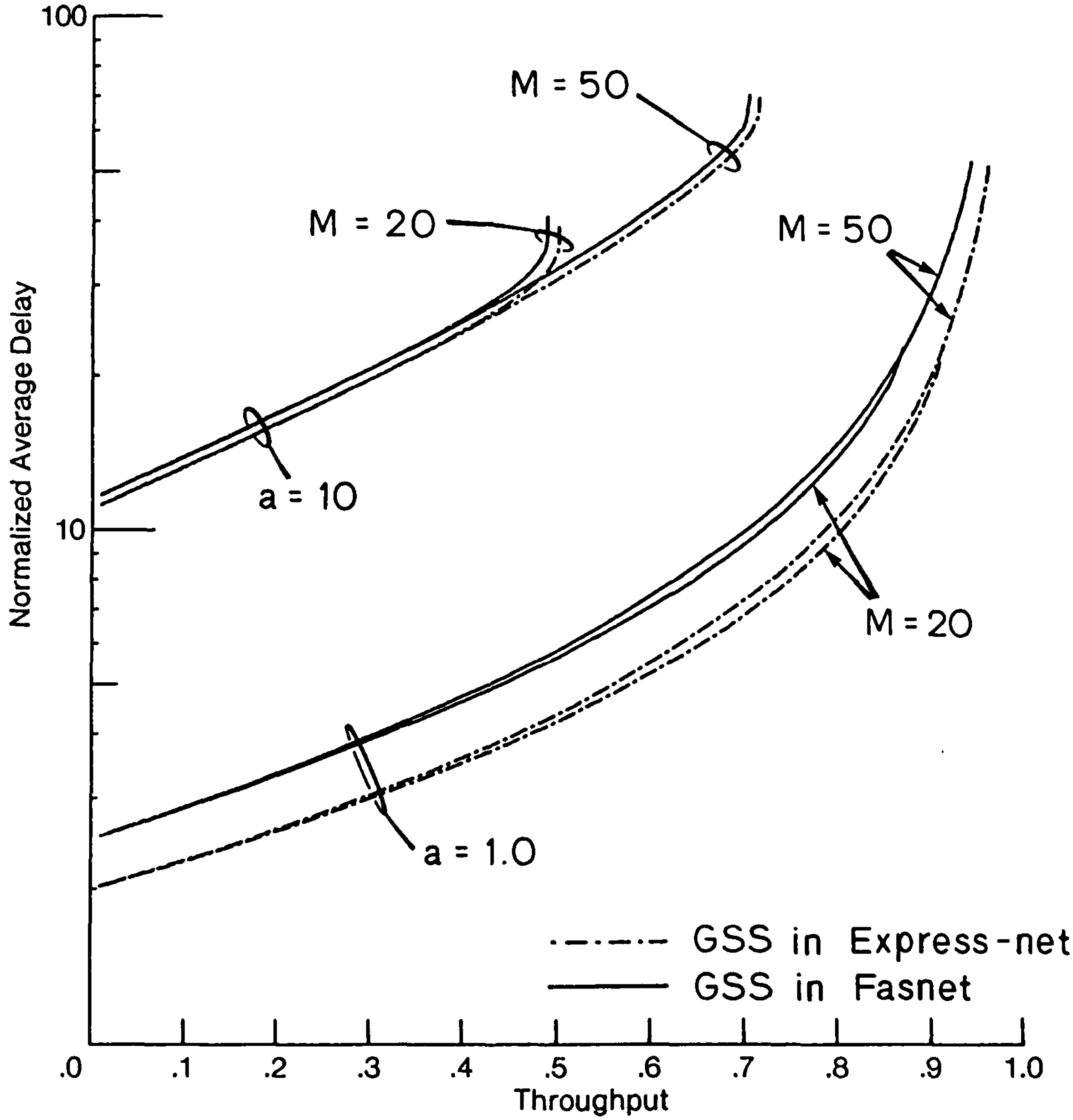


Fig. 19 Average delay normalized by the packet transmission time $T$ versus the channel throughput for GSS in Express-net as compared to GSS in Fasnet.

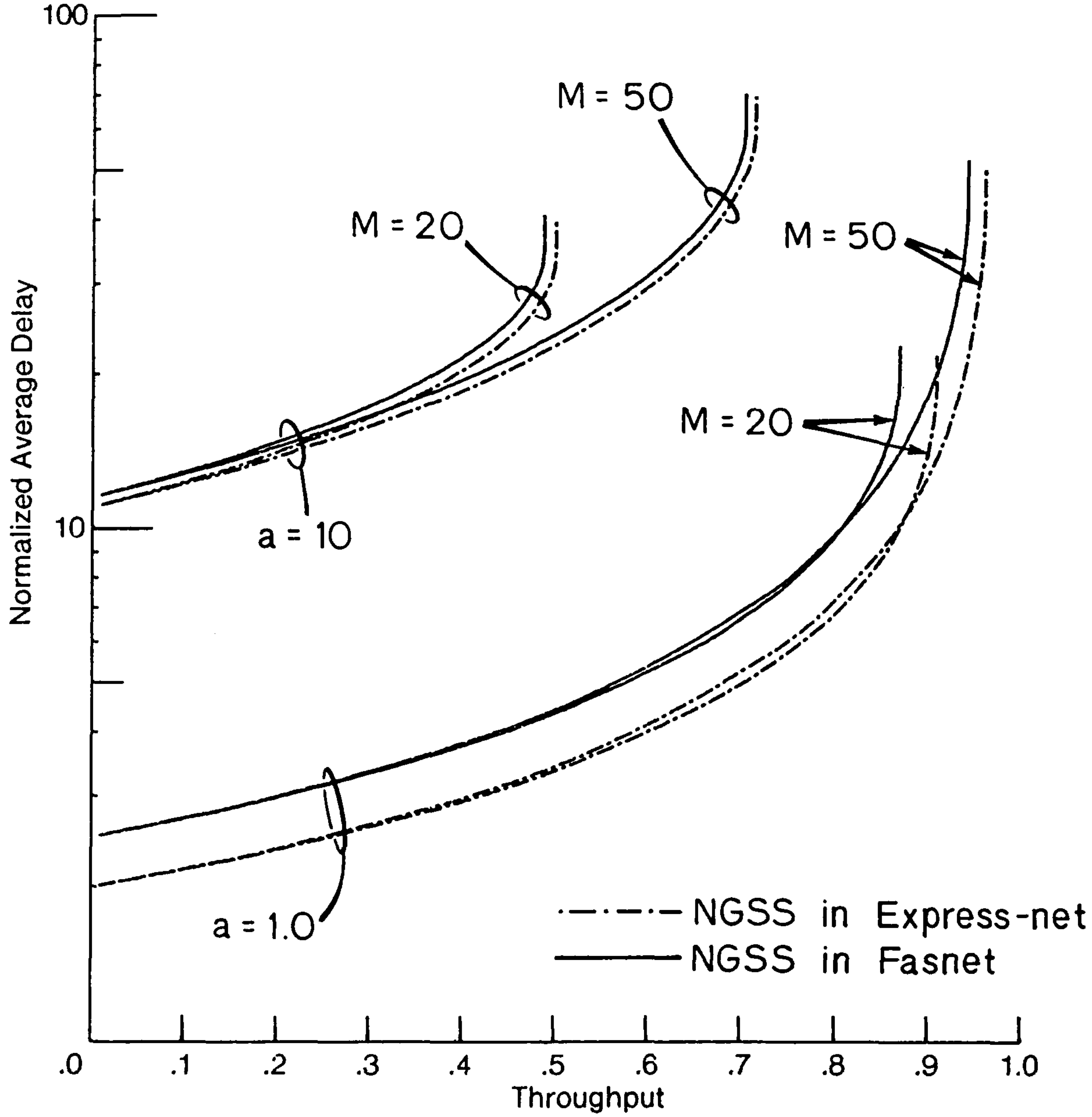


Fig. 20 Average delay normalized by the packet transmission time $T$ versus the channel throughput for NGSS in Fasnet as compared to NGSS in Express-net.

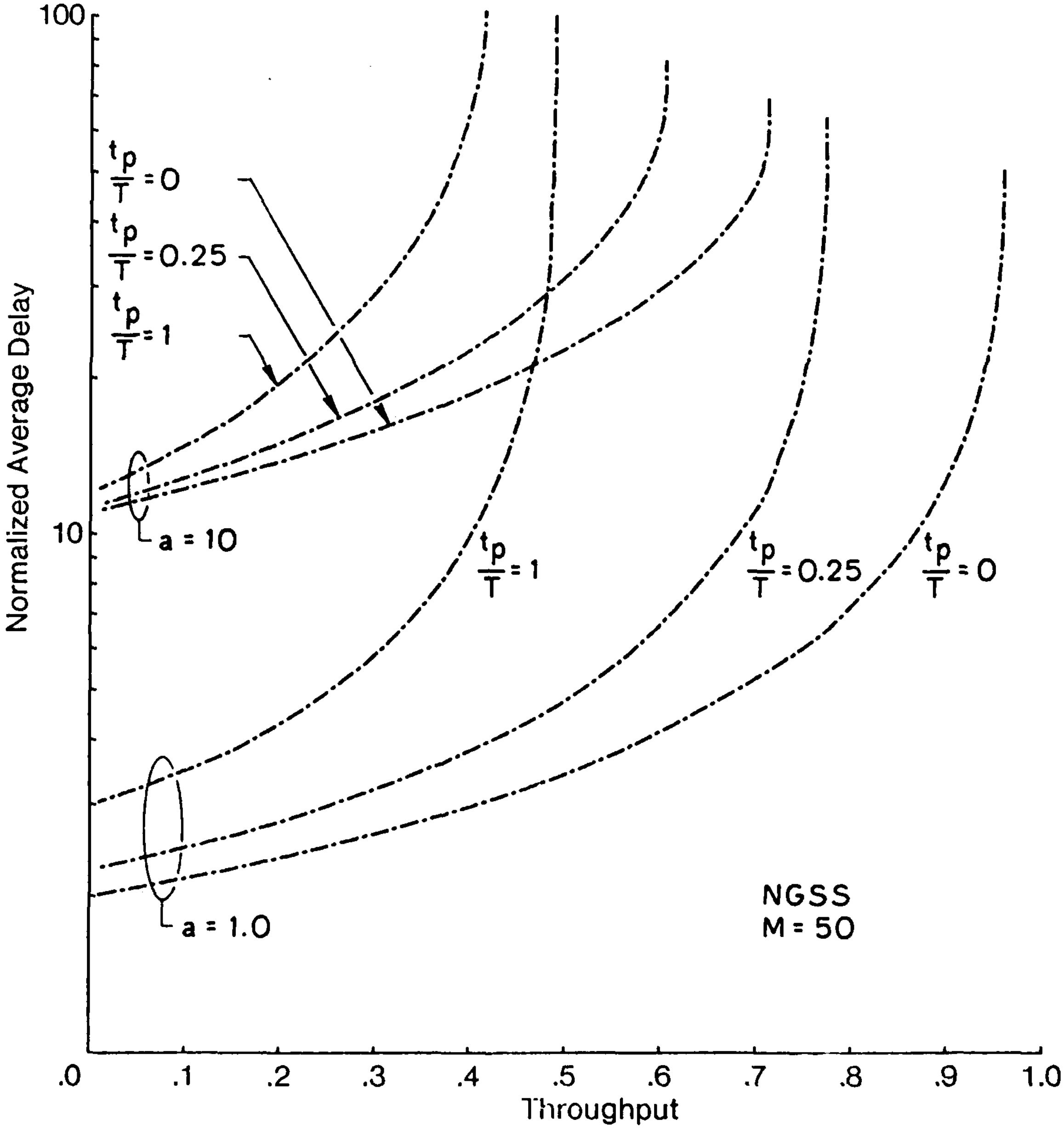


Fig. 21 Average delay normalized by the transmission time $T$ versus the channel throughput showing the effect of the preamble on the throughput-delay performance of NGSS in Express-net.

The results presented above were obtained from the mean value analysis and represent the average performance over all users. For Express-net with the NGSS discipline, service is offered to each user when it sees the EOT (either on the outbound or inbound channel). Therefore the EOT can be viewed as an implicit token passed from each user to the next in sequence. Due to the symmetry of this organization, the system is fair and all users achieve the same performance. In GSS and MUFS on the other hand, the synchronizing event is the beginning of a slot which always sweeps the channel from the most upstream user to the most downstream user. As will be seen in the results discussed below, this mode of operation favors the upstream users by giving channel access to the most upstream of all the users contending for a given slot. In the distribution of delay analysis of GSS and MUFS we derived the performance achieved by each user. This enables us to determine the extent to which this performance is affected by the user's location on the network.

First we consider GSS. In Figs. 22, 23 and 24 we show $M$ times the throughput achieved by the most upstream user and the most downstream user as a function of $M\lambda$, for various values of $M$ and $a$. We refer to these two users as user 1 and user $M$ respectively. The curves show that initially, $S$ increases as $M\lambda$ increases from zero. At low loads there are long idle periods between packet generations, rounds are short, and the throughput achieved by each user is not sensitive to its position on the network. As the network capacity is approached, we see that user 1 achieves a significantly higher throughput than user $M$. This occurs since in any given round, user 1, having transmitted its packet at the beginning of the round, has the remainder of that round in which to generate a new packet before the next SOC. User $M$ on the other hand, having transmitted at the end of the round, has only the inter-round overhead period before the next SOC in which to generate its new packet and thus is less likely to be ready at the beginning of the next cycle. As $M$ increases the difference in $S$ between these two users increases. For large values of $a$ this difference is not as pronounced as for small $a$ due to the fact that the inter-round overhead becomes the dominant factor affecting the performance results and

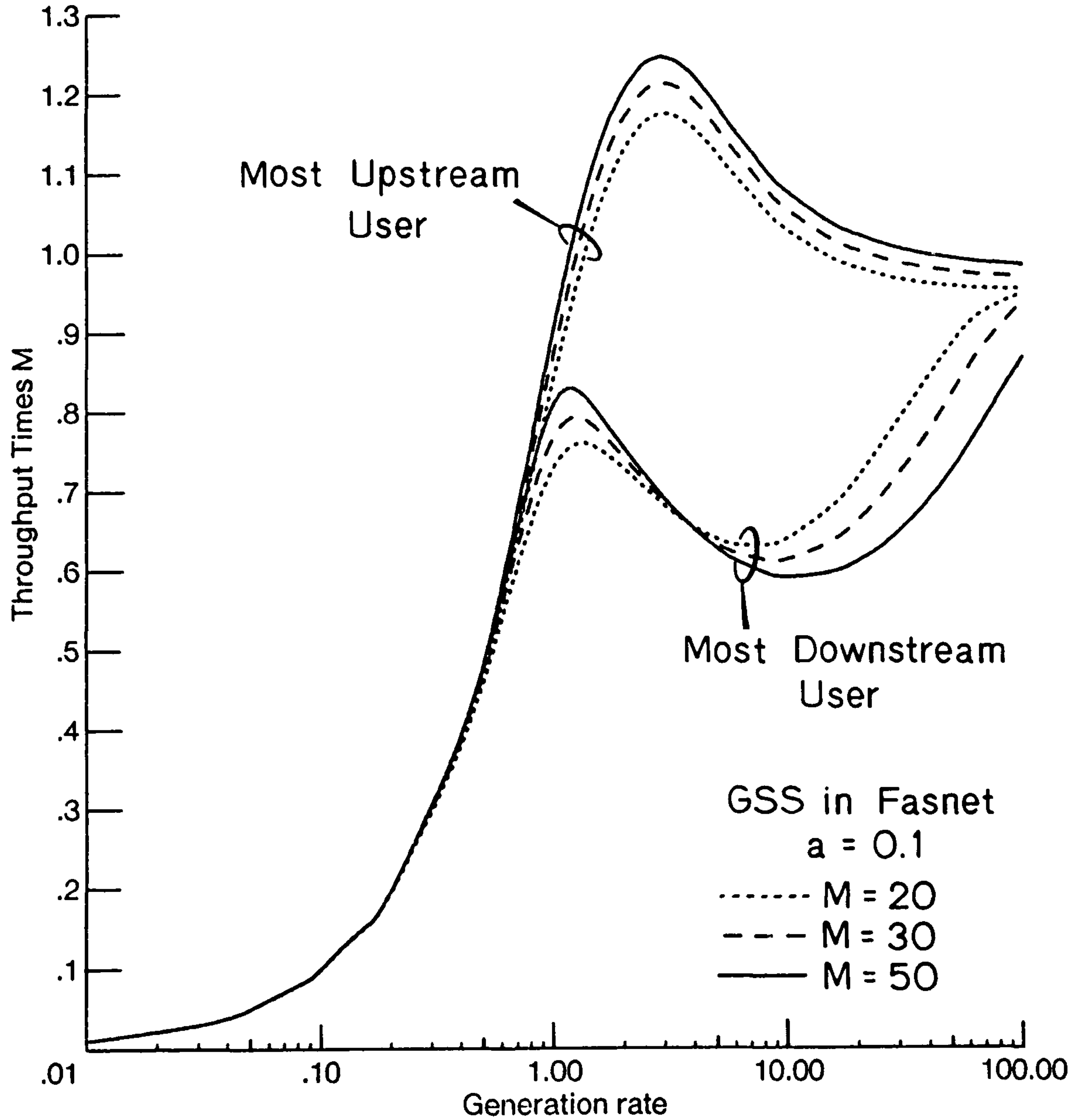


Fig. 22 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for GSS in Fasnet as achieved by the most upstream user and the most downstream user for $a = 0.1$.

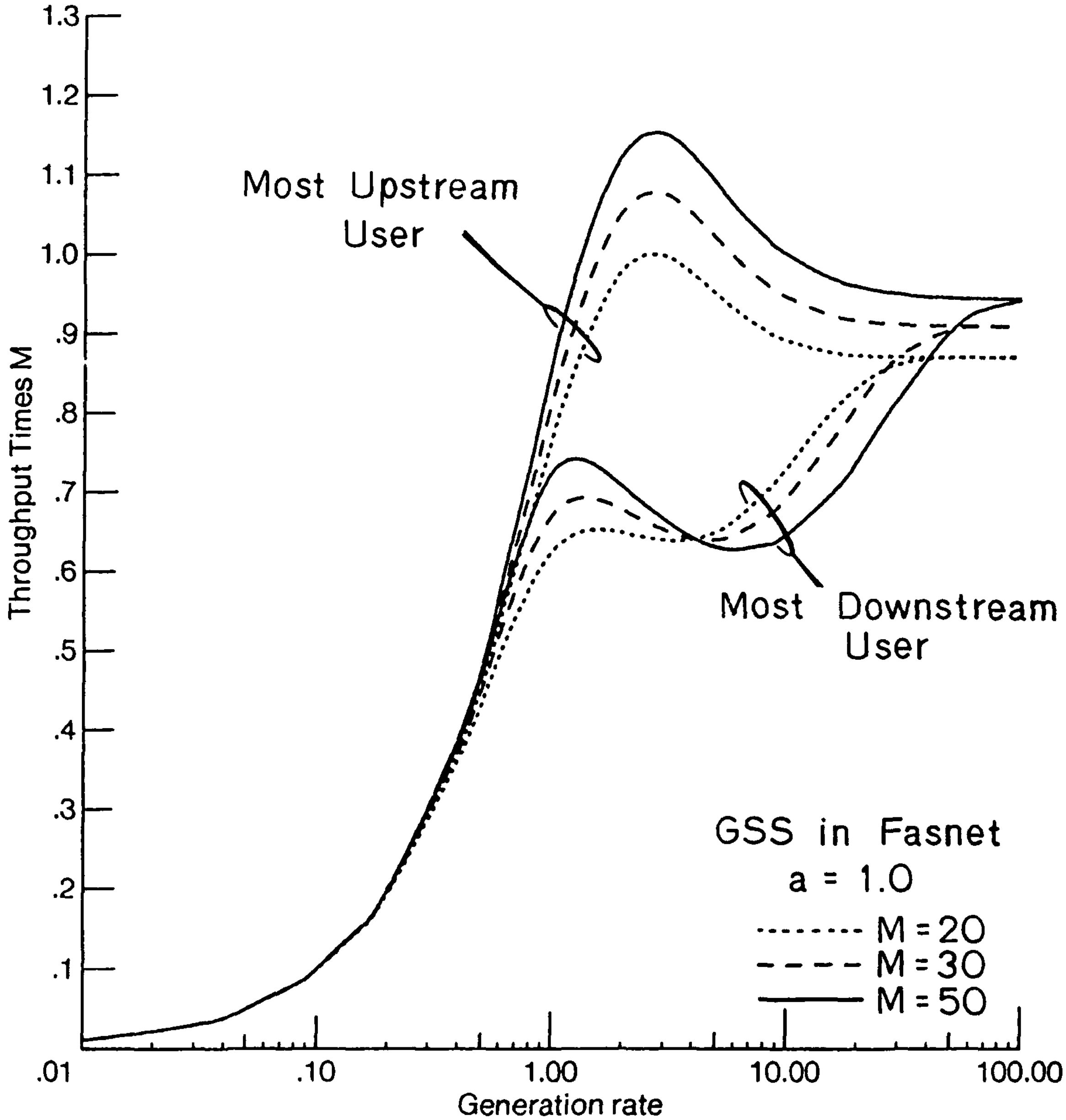


Fig. 23 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for GSS in Fasnet as achieved by the most upstream user and the most downstream user for $a = 1.0$.

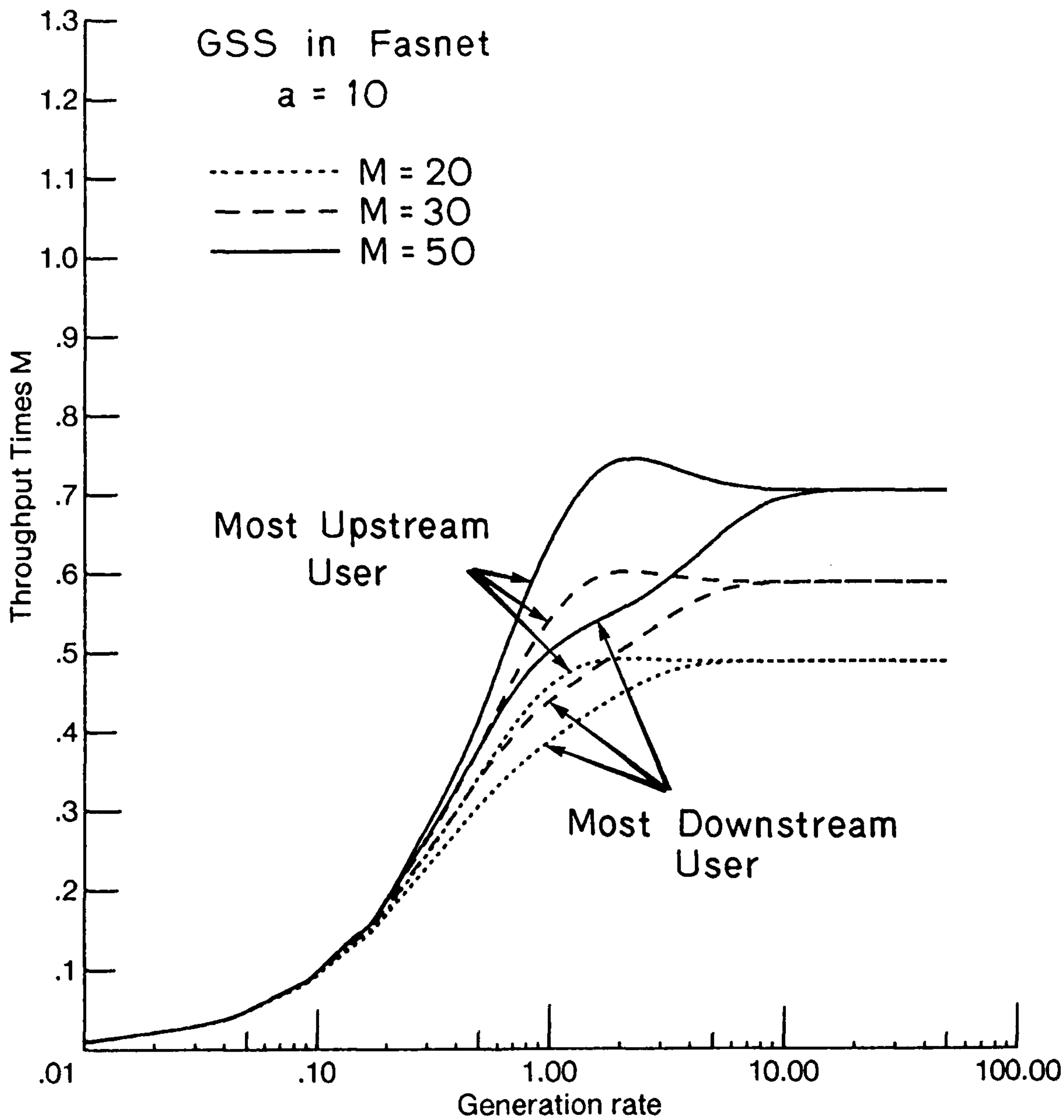


Fig. 24 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for GSS in Fasnet as achieved by the most upstream user and the most downstream user for $a = 10$.

its effect is the same on all users. Finally, as $M\lambda \rightarrow \infty$, user $M$ will generate a new packet during the inter-round gap with probability 1 assuming that $a > 0$; hence user 1 and user $M$ will achieve the same throughput which is given by the network capacity divided by $M$. In the limiting case where $a = 0$, user $M$, having transmitted at the end of a given round, will be ready at the beginning of the next round with probability 0; in particular, at $\lambda = \infty$, user $M$ will transmit once in every two rounds and achieve a throughput of only half that achieved by the other users. The throughput achieved by any of the other users lies within the bounds of user 1 and user $M$. In fact, any given user will achieve a throughput which is greater than any user downstream from it and less than any user upstream from it. The throughput times $M$ for each user in a network with $a = 1.0$ and $M = 10$ is shown in Fig. 25. Recall that each user has only a single buffer. If, however, a multiple packet buffer is provided then a user could generate additional packets for transmission before transmitting the one at the front of the queue. This would reduce the extent of the unfairness suffered by those users on the downstream side of the network. In the limiting case where each user had an infinite buffer, all of the users would achieve the same throughput assuming that they were all generating packets at the same rate.

For the MUFS discipline, the throughput is plotted as a function of $M\lambda$ for various values of $a$ and $M$ in Figs. 26, 27 and 28, and exhibits the same characteristics as for GSS. Note that for MUFS there is not as much of a discrepancy in the service achieved by the individual users.

It is important to point out that for Fasnet, there are two separate channels on which users can transmit data. In the analysis we consider only one of these and, in addition, we assume that all users are generating packets for this channel at the same rate. In actual fact the downstream users on a given channel will most probably require a lower throughput on this channel than the upstream ones since they will be transmitting mostly on the other channel. In fact, the most downstream user on a given channel will not

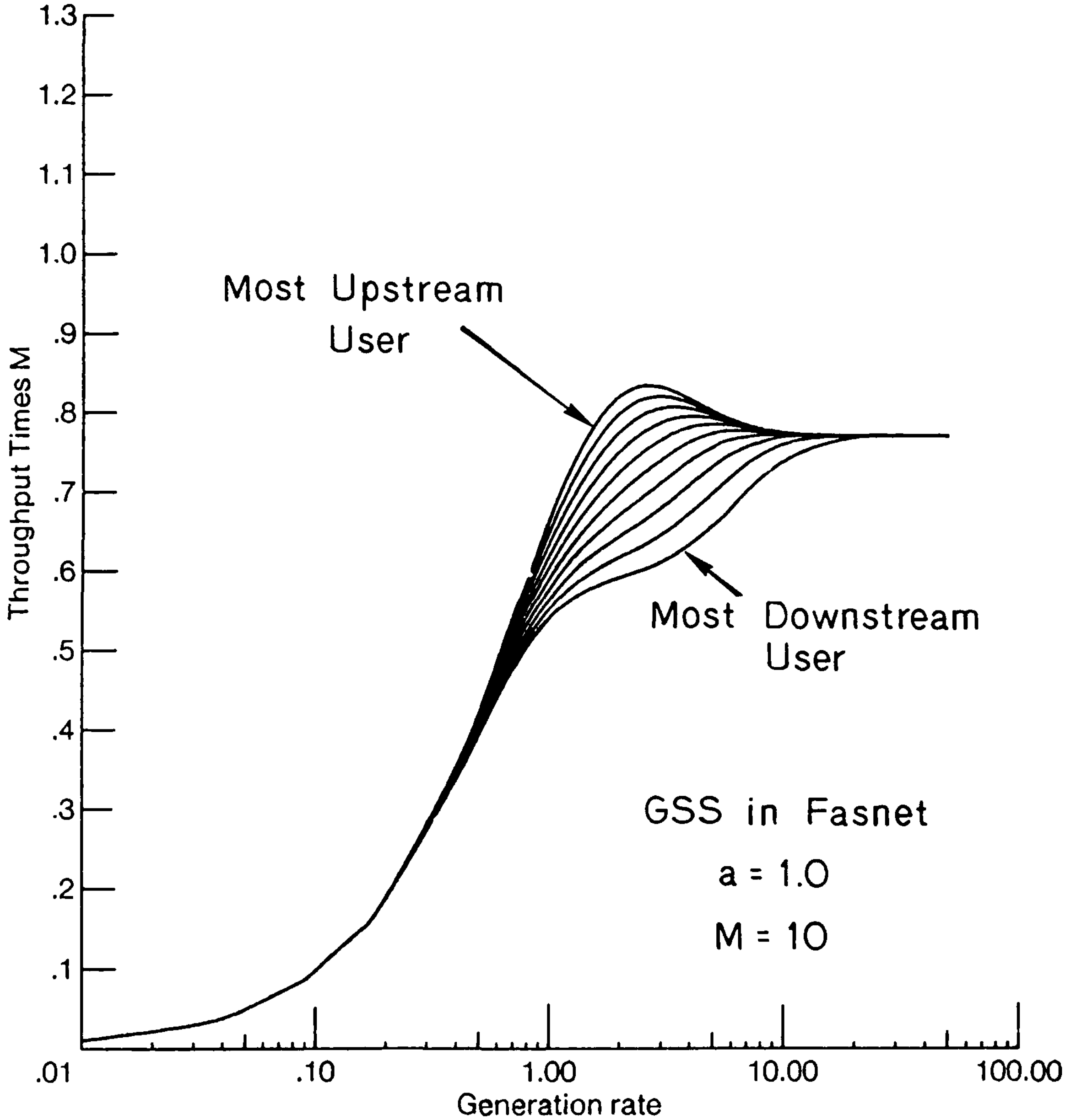


Fig. 25 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for GSS in Fasnet with $a = 1$ and $M = 10$ as achieved by each user on the network.

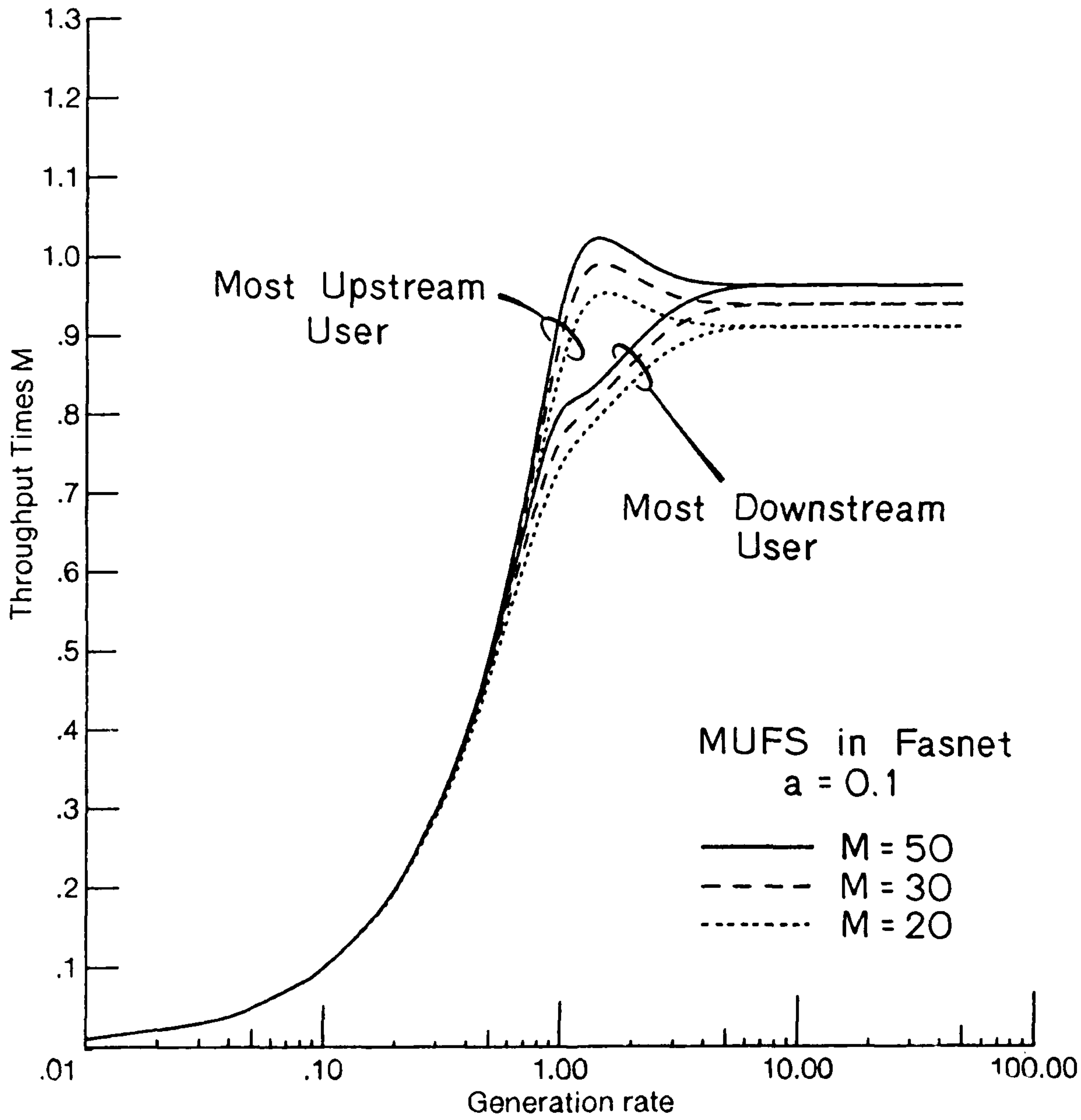


Fig. 26 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for MUFS in Fasnet as achieved by the most upstream user and the most downstream user for $a = 0.1$.

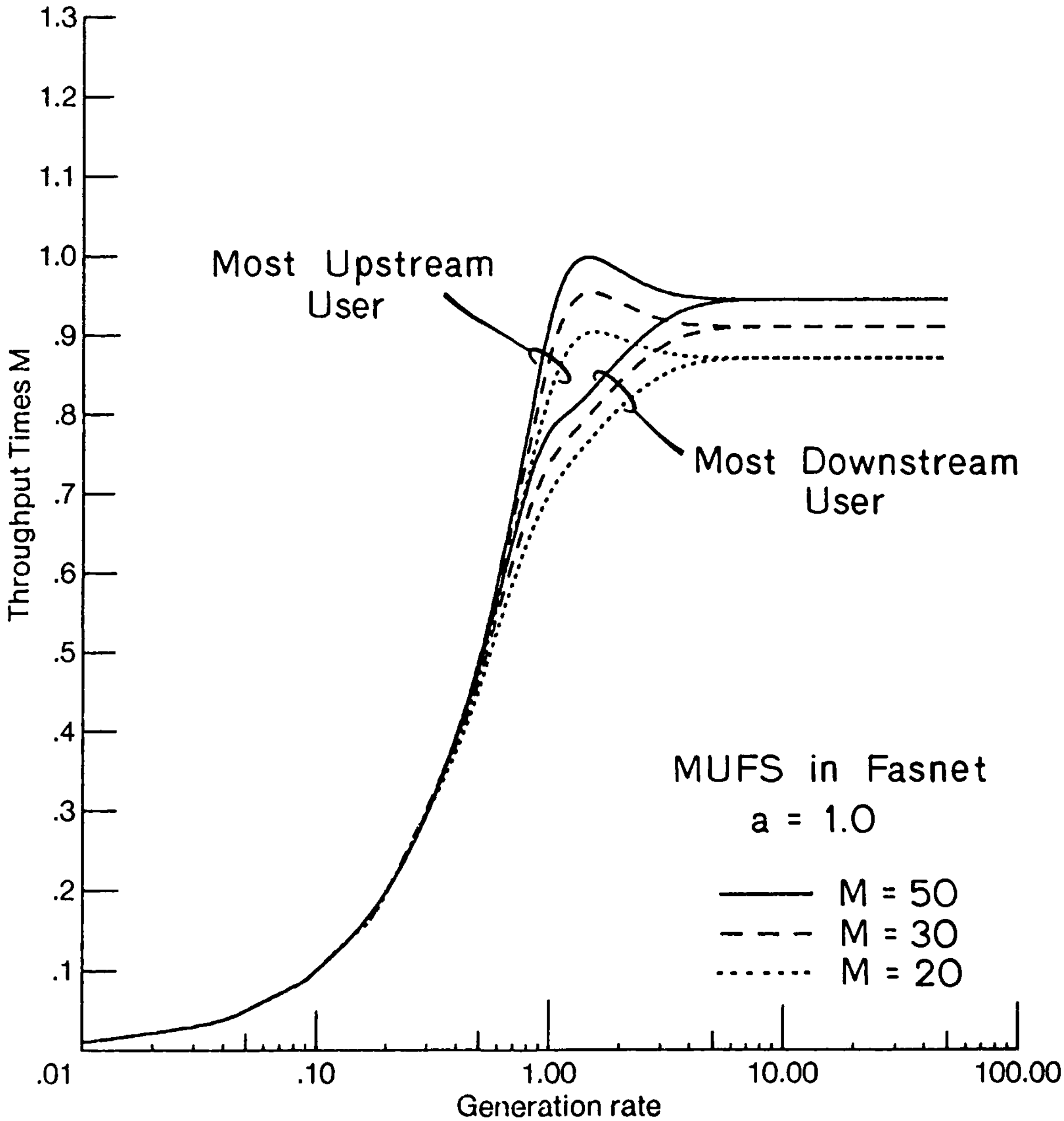


Fig. 27 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for MUFS in Fasnet as achieved by the most upstream user and the most downstream user for $a =$ 1.0.

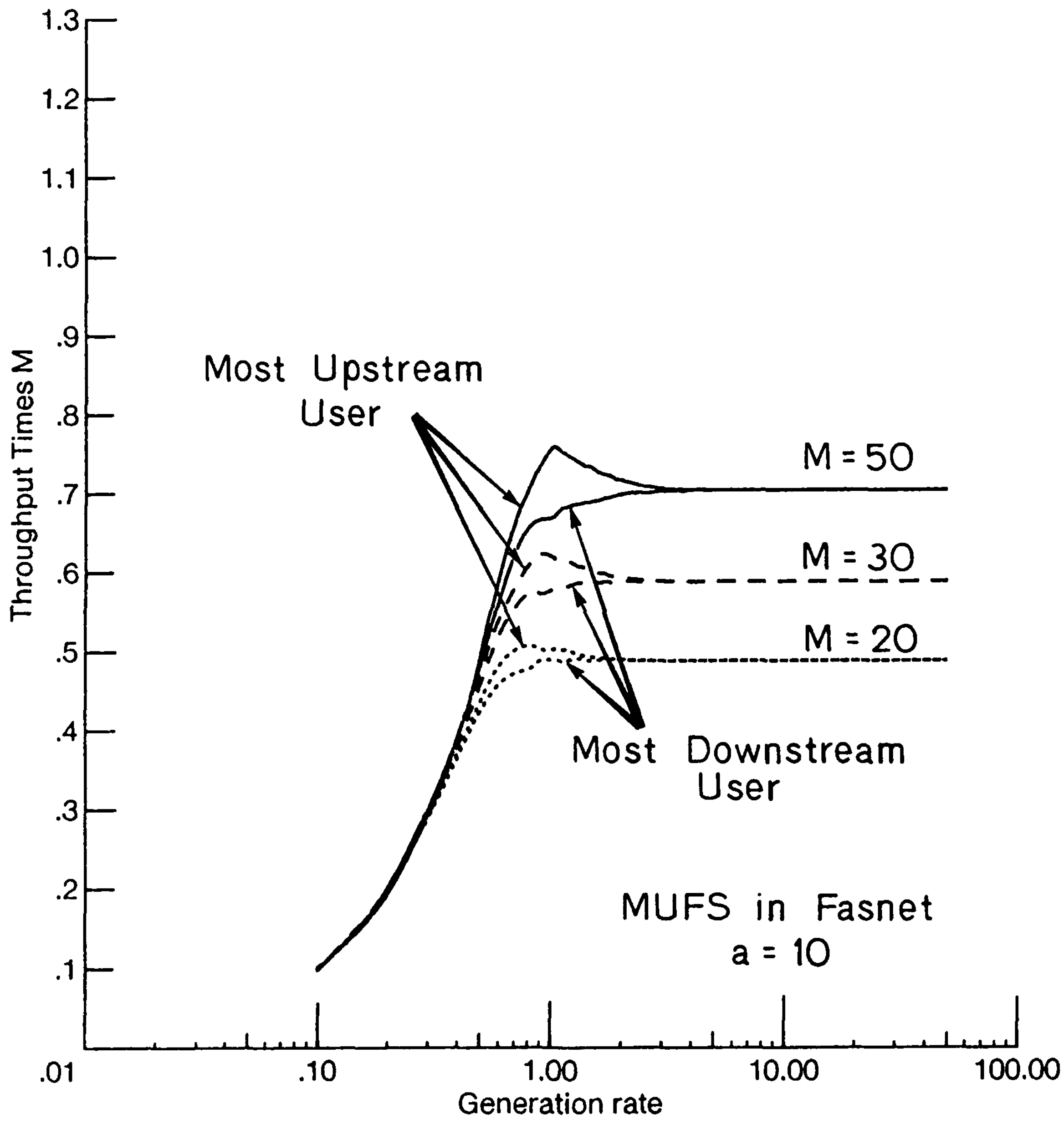


Fig. 28 Throughput multiplied by $M$ versus the generation rate $M\lambda T$ for MUFS in Fasnet as achieved by the most upstream user and the most downstream user for $a = 10$. These curves were obtained by simulation.

transmit any packets on that channel since there is nobody further downstream to receive it.

The difference in average delay between the most upstream and the most downstream users in Fasnet is shown for GSS in Fig. 29, and for MUFS in Fig. 30. Since in a given round user 1 is serviced before user $M$, it achieves a lower delay for a given $S$. It is interesting to note that in GSS the delay of user 1 is bounded from above by the maximum length of a cycle which is $MT + Y$. For user $M$ the delay is bounded by twice the maximum length of a round plus an inter-round overhead period, that is $2MT + Y$, even though at saturation ($\lambda \rightarrow \infty$) the delay will be $MT + Y$. In MUFS and also NGSS the delay of a packet from any user is always bounded by $MT + Y$. If Express-net were to be operated under the GSS discipline then, in this system too, the average delay achieved by any user would be dependent on that users location on the network. There would however be a slight improvement in the overall throughput-delay characteristics as compared to the throughput-delay characteristics achieved in Fasnet under GSS, due to the smaller inter-round overhead for a given $a$ in Express-net. In Fig. 31 we plot the throughput-delay trade-off as achieved by user 1 and user $M$ in both Express-net and Fasnet operating under the GSS discipline, which shows the similarity of their characteristics.

Finally, we examine the variance of delay. The relationship between the variance and the throughput for each of the three service disciplines is shown in Figs. 32, 33, and 34 for $a = 1.0$ and 10 and for various values of $M$. For GSS and MUFS we show the variance of delay versus $S$ as achieved by user 1 and user $M$. Since for NGSS all users achieve the same performance, we show the variance versus $S$ as achieved by any user on the network. For $S = 0$ the variance is non-zero due to the randomness between the time of arrival of a packet and the time at which the user may transmit this packet. Depending on the service discipline, the time at which a user may transmit may be after the next locomotive or after the next SOC in the case of NGSS or GSS, or at the beginning of the next slot in the case

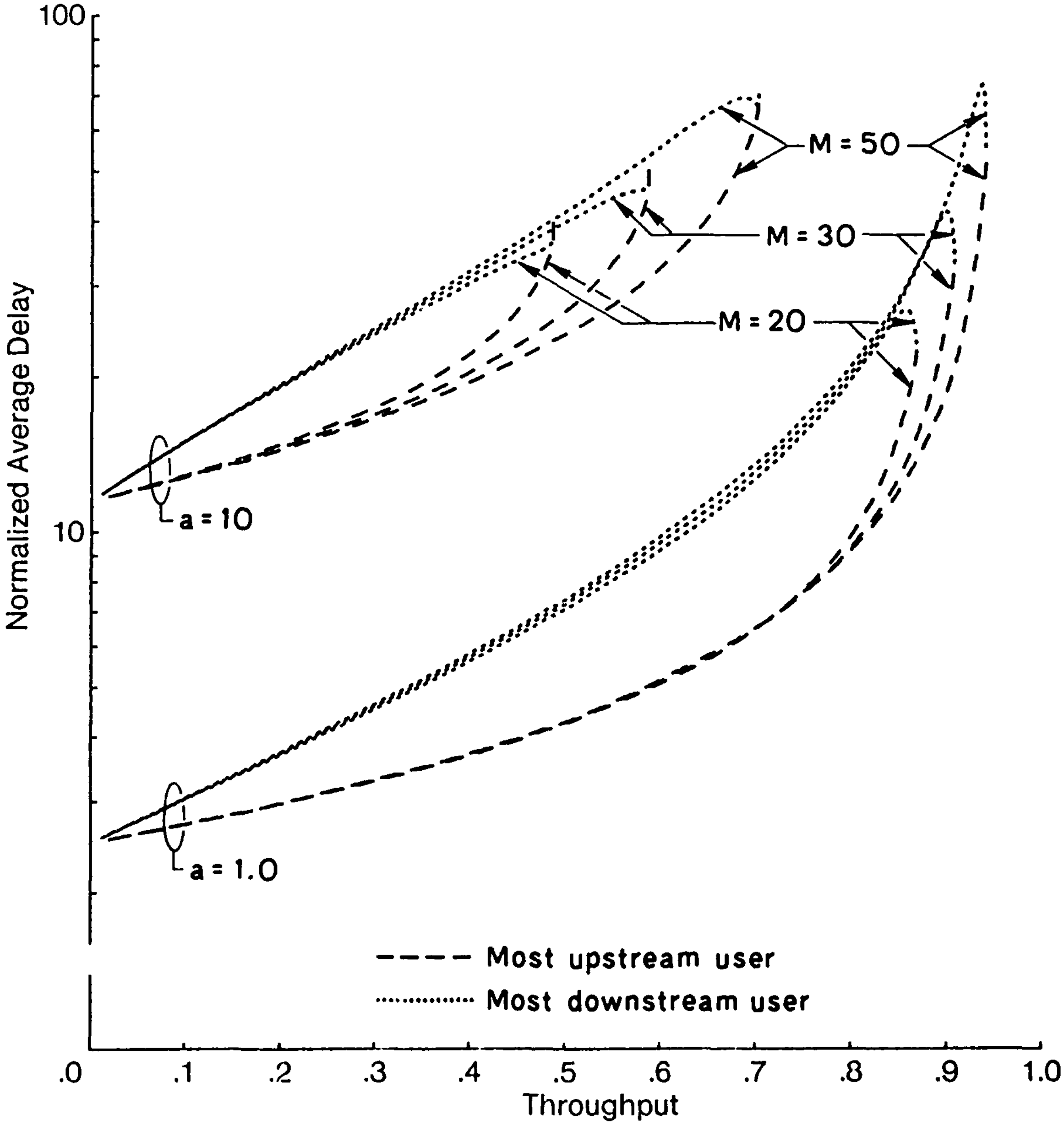


Fig. 29 Normalized average delay versus throughput for GSS in Fasnet as achieved by the most upstream user and the most downstream user.

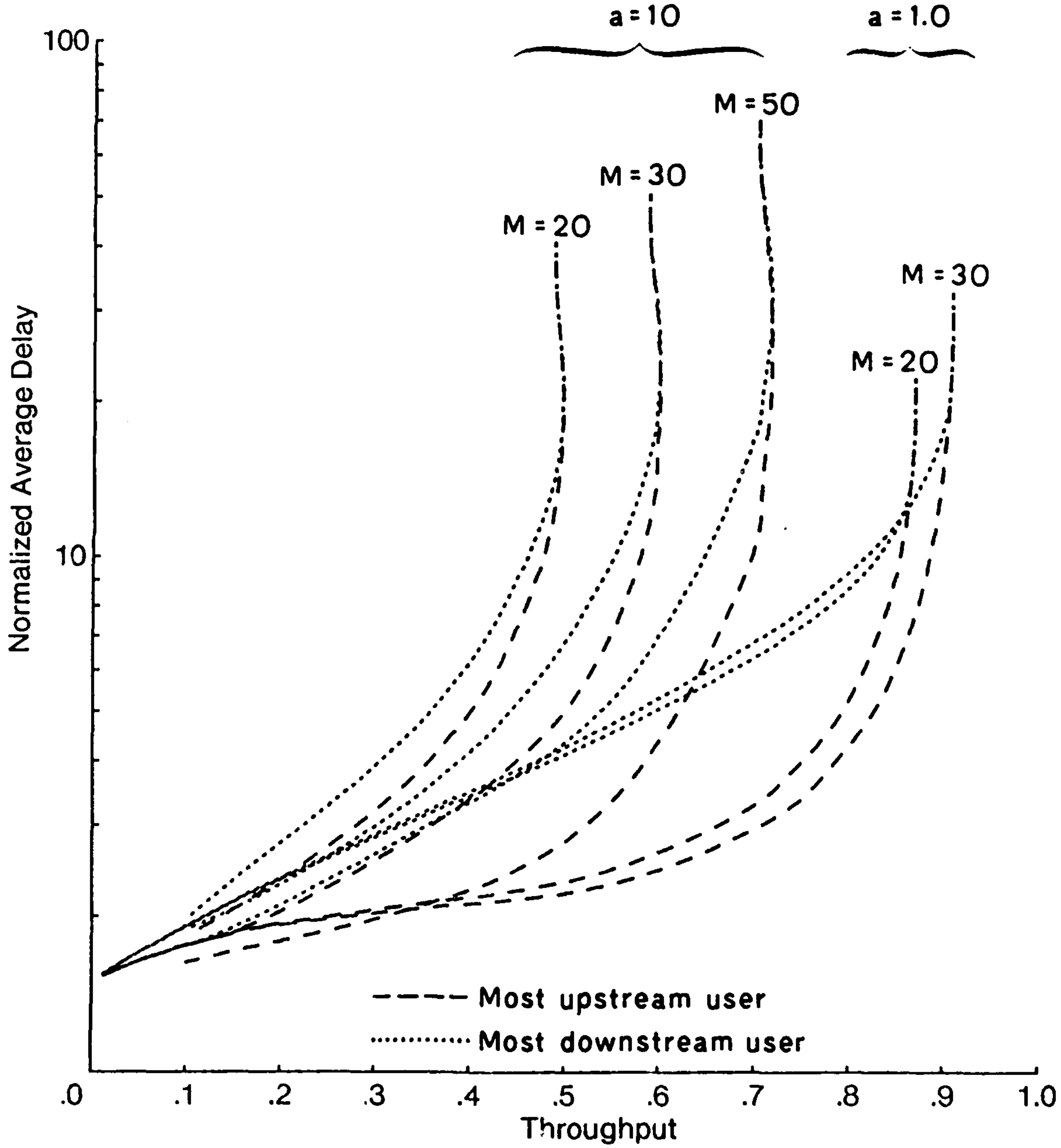


Fig. 30 Normalized average delay versus throughput for MUFS in Fasnet as achieved by the most upstream user and the most downstream user.

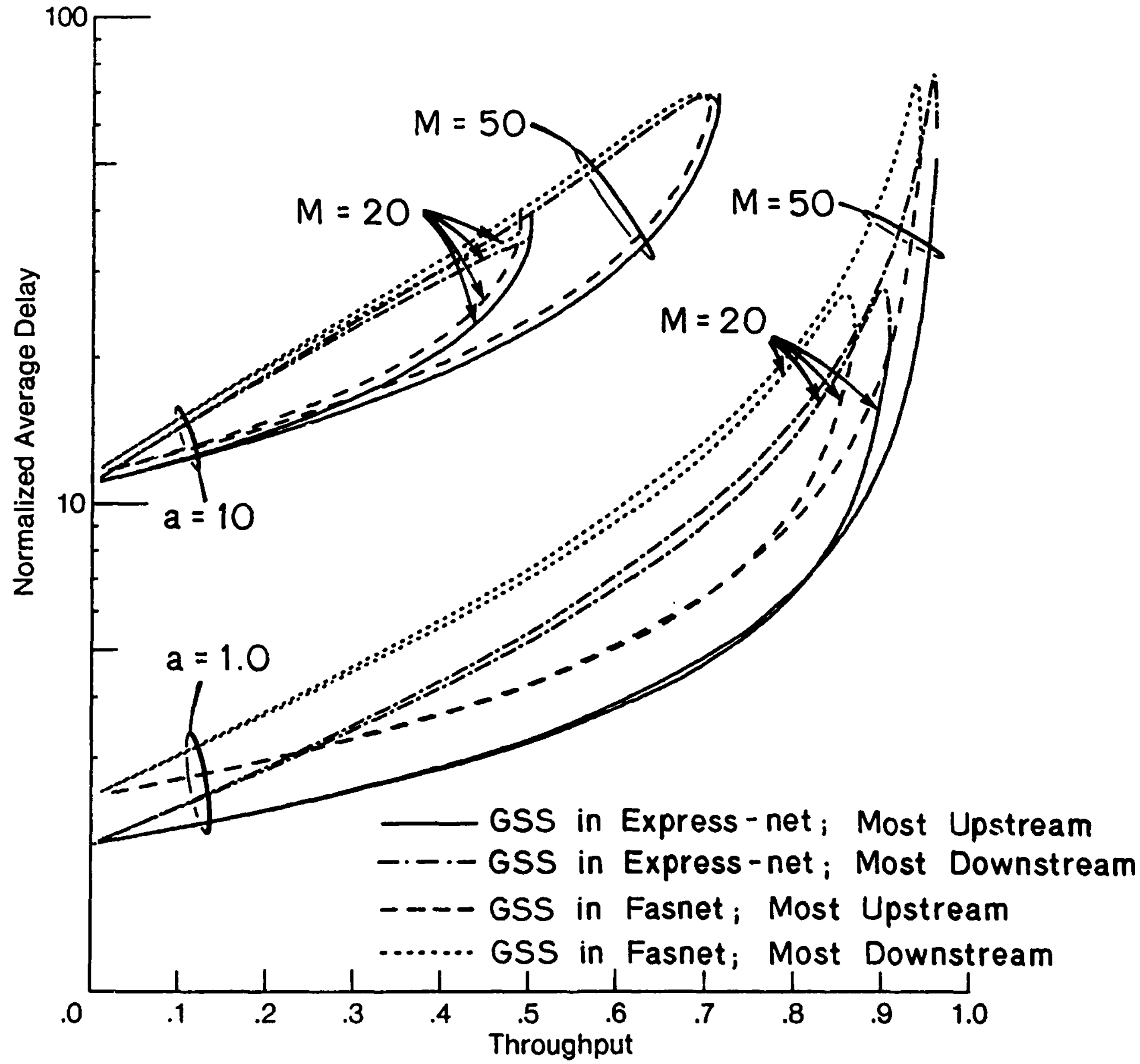


Fig. 31 Comparison of the throughput-delay characteristics as achieved by the most upstream user and the most downstream user in both Express-net and Fasnet operating under the GSS discipline.

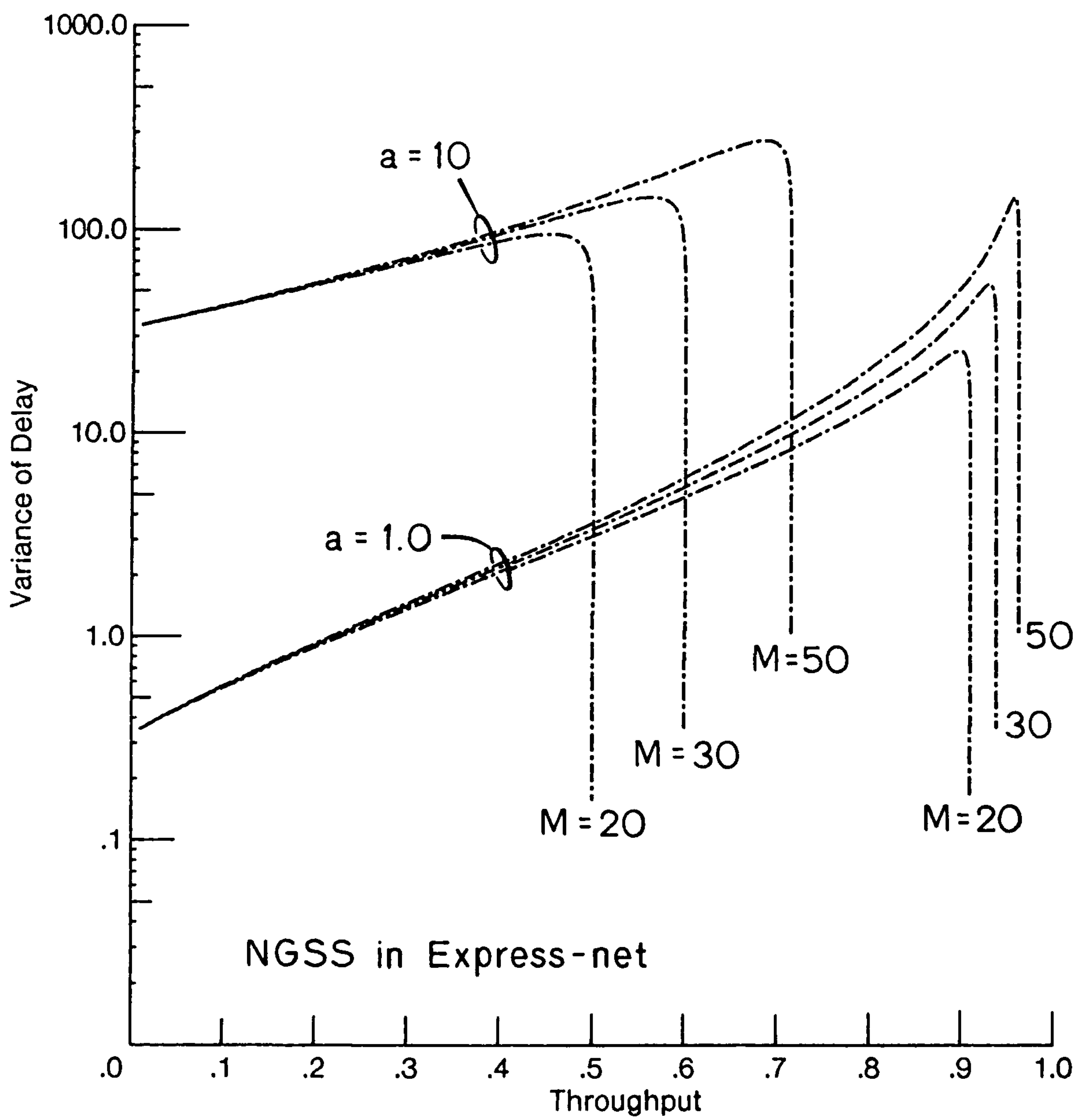


Fig. 32 Variance of delay versus throughput for NGSS in Express-net. The curves shown are for the variance as achieved by any user on the network.

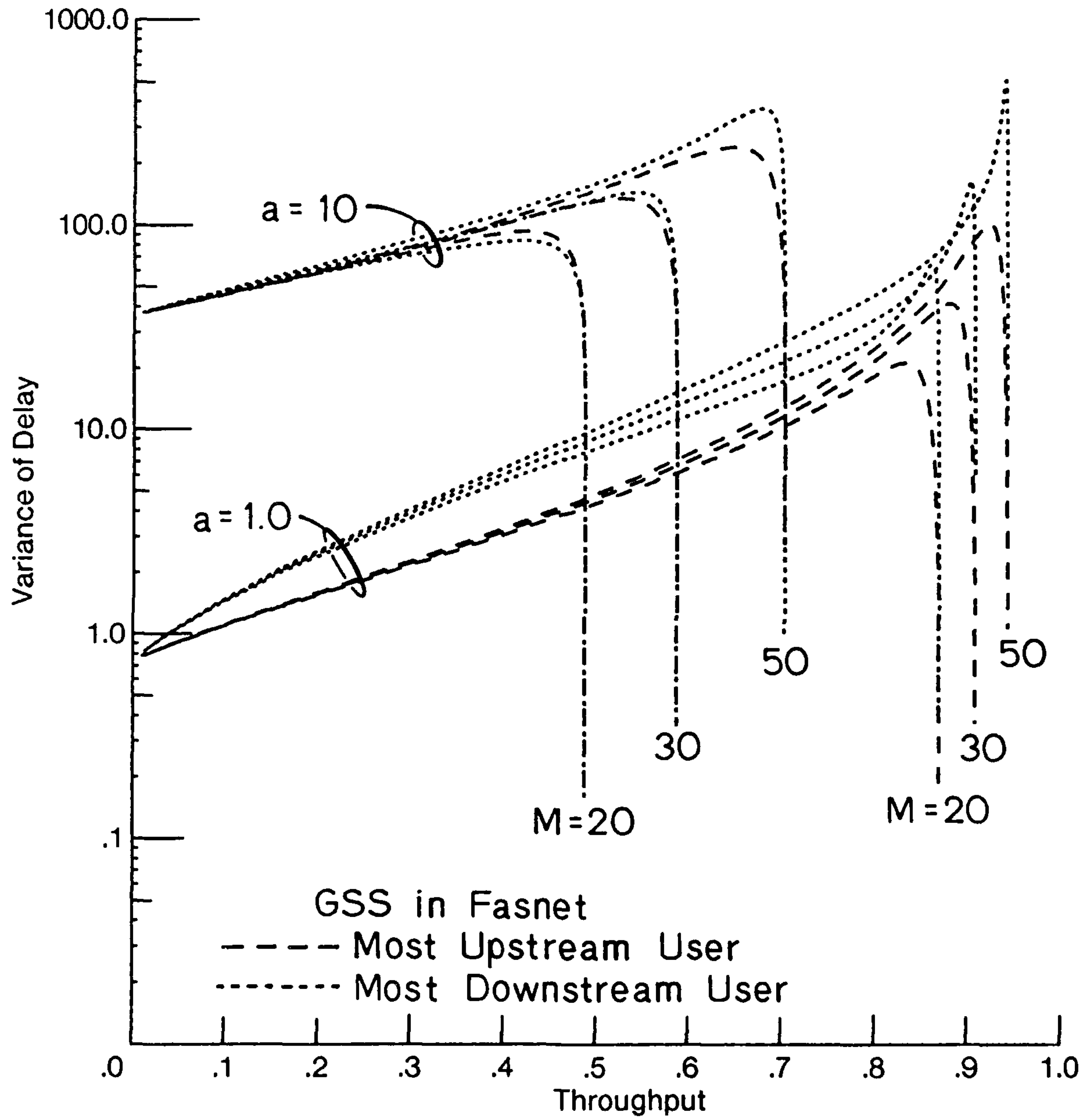


Fig. 33 Variance of delay versus throughput for GSS in Fasnet. The variance as achieved by the most upstream user and the most downstream user is shown for each of the values of $a$ and $M$.

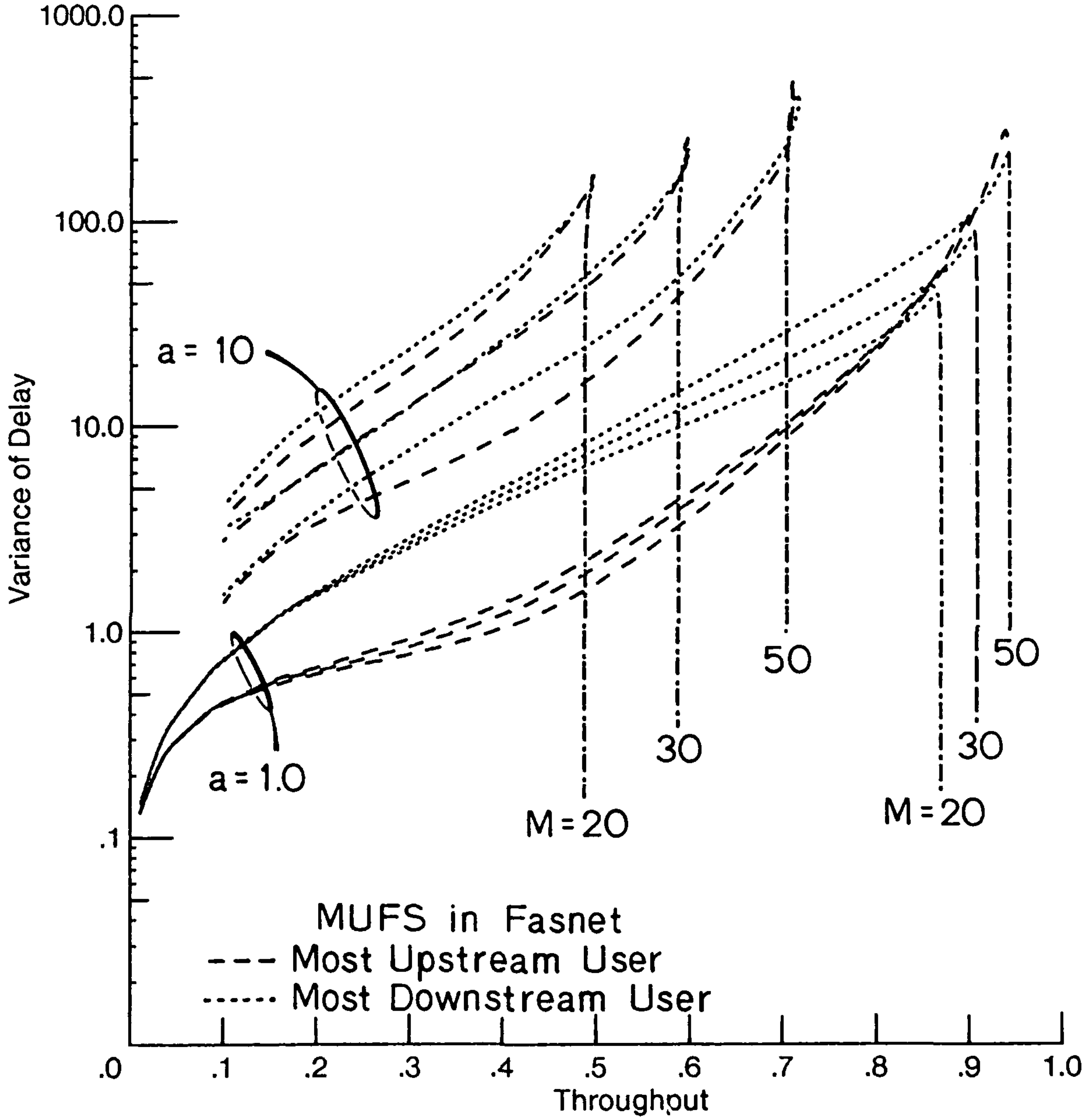


Fig. 34 Variance of delay versus throughput for MUFS in Fasnet. The variance as achieved by the most upstream user and the most downstream user is shown for each of the values of $a$ and $M$. The curves shown for $a = 10$ were obtained by simulation.

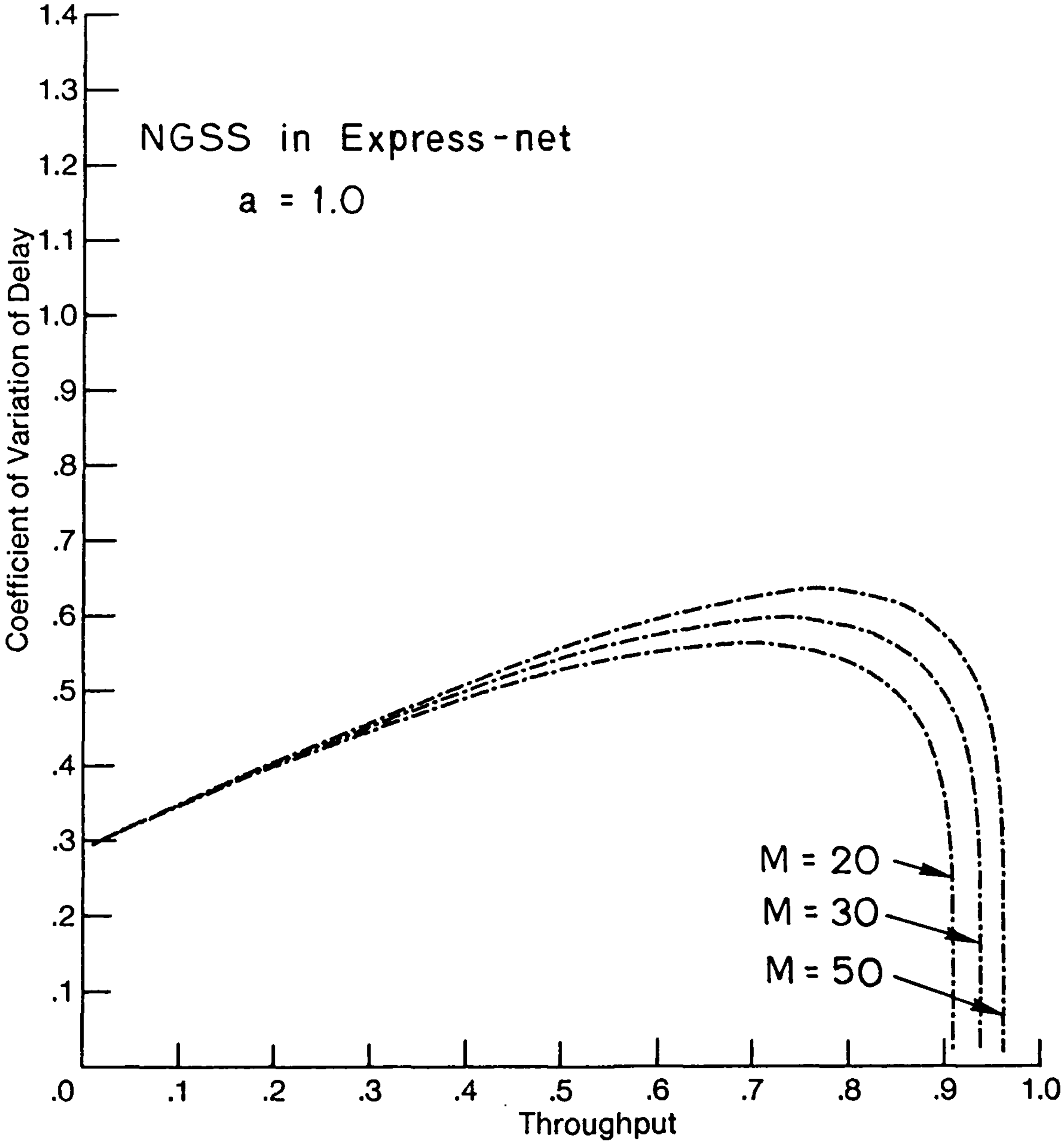


Fig. 35 Coefficient of variation of delay versus throughput for NGSS in Express-net with $a = 1.0$. The curves shown are for the variance as achieved by any user on the network.

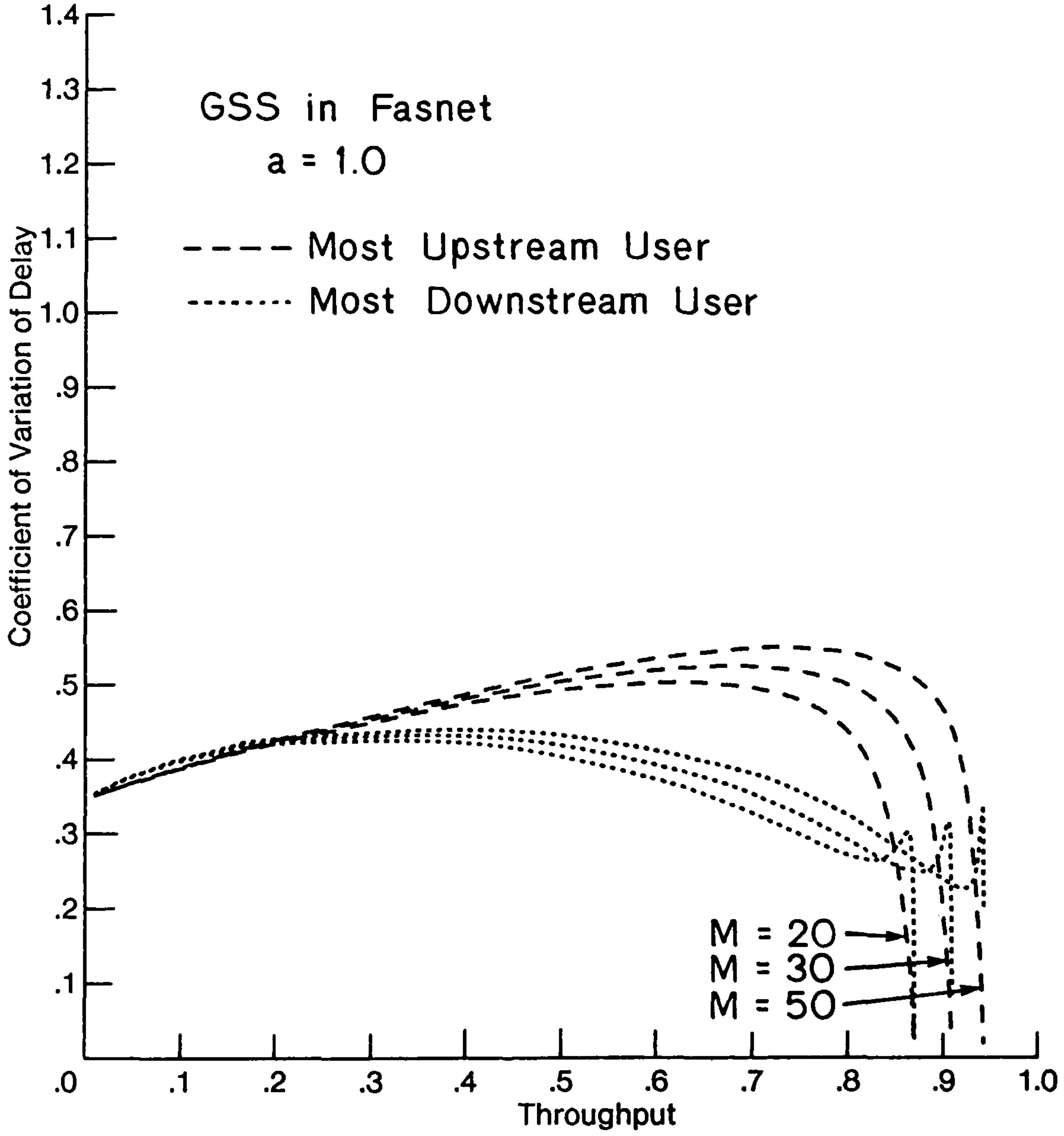


Fig. 36 Coefficient of variation of delay versus throughput for GSS in Fasnet with $a = 1.0$. The variance as achieved by the most upstream user and the most downstream user is shown for each of the values of $M$.

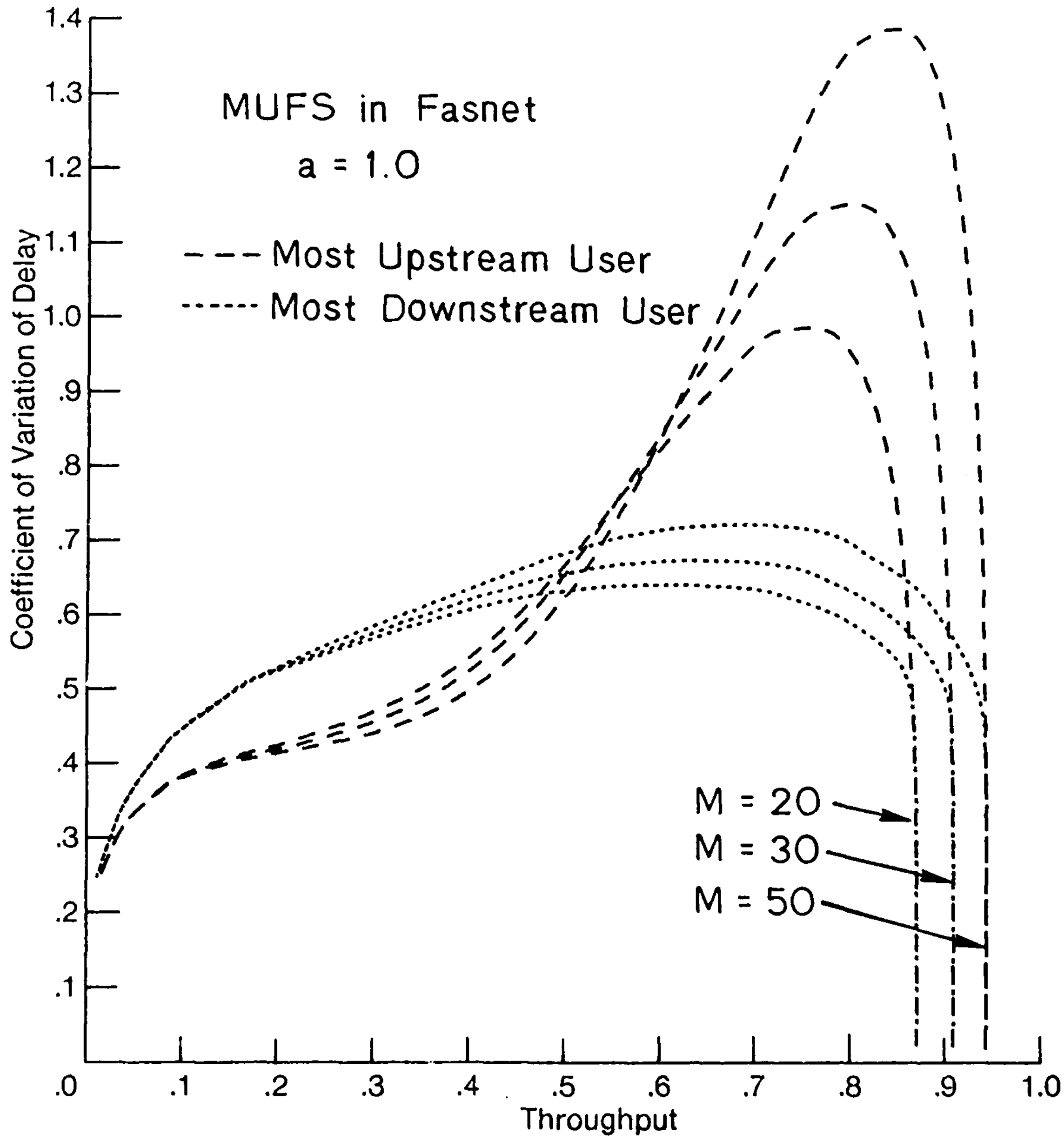


Fig. 37 Coefficient of variation of delay versus throughput for MUFS in Fasnet with $a = 1.0$. The variance as achieved by the most upstream user and the most downstream user is shown for each of the values of $M$.

of MUFS. This implies that, for large $a$, the variance for MUFS at low $S$ is lower than for GSS and for NGSS since the randomness in the packet delay in this case is associated with the time of arrival taking place within a slot which is shorter than the period separating two consecutive locomotives or SOCs. As $\lambda \rightarrow \infty$, the variance drops to zero since at each user, a new packet is generated as soon as the previous one is transmitted, all rounds are of full length and the packet delay is deterministic and equal to $MX + Y$. It is interesting to note that the variance incurred is highest for $S$ close to the network capacity while the variance is zero at the network capacity. The coefficient of variation of delay as a function of the throughput in shown for each of the three service disciplines in Figs. 35, 36, and 37 for $a = 1.0$ and various values of $M$. We noted that, for all the cases examined, the coefficient of variation for GSS and NGSS is always less than one while for MUFS it may be greater than 1.

## 6. Conclusion

In this report we investigated the performance of two Unidirectional Broadcast Systems that have been presented in the literature; Express-net and Fasnet. Two versions of the access protocol have been presented for Fasnet. From these two protocols and the one for Express-net, three service disciplines were identified which we called Non Gated Sequential Service (Express-net), Gated Sequential Service (Fasnet) and Most Upstream First Service (Fasnet). In addition we noted that, with a simple change in their respective access protocols, Express-net could be operated in GSS mode and Fasnet could be operated in NGSS mode. However only Fasnet could support MUFS.

From the analyses of these service disciplines numerical results were computed. We showed that these systems, unlike random access techniques, can achieve a channel utilization close to 100 percent even when the channel bandwidth is high or the propagation delay of the signal over the network is large. In addition, the network remains stable as the load increases to infinity without the need for any dynamic control of the access protocol. The throughput delay characteristics are excellent and the maximum delay is bounded from above by a finite value which is easily computed. As the throughput approaches the network capacity the variance of delay reaches a peak and then drops to zero. At network capacity the system becomes deterministic with all users transmitting in every round.

Finally, we noted that all three service disciplines exhibit similar performance characteristics. However, in GSS and MUFS there is an element of unfairness which favors some users over others depending on their location on the network, while for NGSS the access protocol is completely fair with all users achieving the same performance.

## References


[1] R. M. Metcalfe and D. R. Boggs, "Ethernet: Distributed packet switching for local computer networks," *Communications of the ACM,* vol. 19, no. 7, pp. 395–403 (1976).

[2] F. Tobagi and V. Bruce Hunt, "Performance analysis of carrier sense multiple access with collision detection," in *Proc. Local Area Commun. Network Symp.*, Boston, MA, May 1979.

[3] L. Kleinrock and F. Tobagi, "Packet switching in radio channels: Part I—Carrier sense multiple access modes and their throughput delay characteristics," *IEEE Trans. Commun.*, vol. COM-23, pp. 1400–1416, Dec. 1975.

[4] F. Tobagi and L. Kleinrock "Packet switching in radio channels: Part IV--Stability considerations and dynamic control in carrier sense multiple access," *IEEE Trans. Commun.*, vol. COM-25, pp. 1103–1120, Oct. 1977.

[5] L. Fratta, F. Borgonovo and F. A. Tobagi, "The Express-net: A local area communication network integrating voice and data", in *Proceedings of the International Conference on the Performance of Data Communication Systems and Their Applications*, Paris France, 14–16 September 1981.

[6] F. Tobagi, F. Borgonovo, L. Fratta, "Express-net: A high-performance integrated-services local area network," *IEEE Journal on Selected Areas in Communications*, vol. SAC-1 Nov. 1983.

[7] J. O. Limb and C. Flores, "Description of Fasnet, a unidirectional local area communications network," *Bell Systems Technical Journal*, September 1982.

[8] John O. Limb, "Fasnet: A proposal for a high speed local network," in *Proc. of Office Inform. Sys. Workshop*, St. Maximin, France, Oct. 1981.

[9] D. J. Farber *et al*, "The Distributed Computing System," in *Proc. of the 7th Annual IEEE Computer Society International Conference* (IEEE Feb. 1973).

[10] F.A. Tobagi and R. Rom, "Efficient round-robin and priority schemes for unidirectional broadcast systems," *Proceedings of the IFIP 6.4 Zurich Workshop on Local Area Networks*, Zurich, Switzerland, August 27-29 1980.

[11] R. Rom and F.A. Tobagi,"Message-based priority functions in local multiaccess communications systems," *Computer Networks*, Vol. 5 No. 4, July 1981, pp. 273-286.

[12] I. Chlamtac, W. Franta and K. D. Levin, "BRAM: The broadcast recognizing access method," *IEEE Trans. Commun.*, Vol. COM-27, No. 8, August 1979, pp. 1183-1190.

[13] A. Roger Kaye, "Analysis of a distributed control loop for data transmission,"

*Proc. Symp. Computer Commun. Networks and Teletraffic,* Polytechnic Institute of Brooklyn, April 1972.

[14] C. Mack, T. Murphy and N. L. Webb, "The efficiency of $N$ machines unidirectionally patrolled by one operative when walking time and repair times are constants," *J. Royal Stat. Soc., Ser. B, 19,* 166-172, 1957.

[15] F. Tobagi et al., "Modeling and measurement techniques in packet communication networks," *IEEE Proceedings,* pp, 1423–1447, Nov. 1978.

[16] F. A. Tobagi and M. Fine, " Performance of unidirectional broadcast local area networks: Express-net and Fasnet." *IEEE Journal on Selected Areas in Communications,* vol. SAC-1, Nov. 1983.

[17] M. Fine and F. A. Tobagi, "Performance of round robin schemes in unidirectional broadcast local networks," in *Proc. of the International Conf. on Communications,* Philadelphia, June 13-17, 1982, pp. 1C.5.1-1C.5.6.

[18] R. Rom, "Performance analysis of the unidirectional broadcast system," in *Proc. of the International Symposium on Local Computer Networks,* IFIP TC-6, Florence, Italy, April 1982.